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Foreword

Math Mammoth Grade 5, International Version, comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 5, only customised for international audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the fifth grade standards in your country.

The international version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses only metric measurement units.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal point, as in the US version.)

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction, and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorising two-dimensional figures (especially triangles);

This book, 5-B, covers more on decimal arithmetic, in chapter 6. The focus is on decimal multiplication and division, and on conversions between measurement units. Chapter 7 has to do with fraction addition and subtraction, and chapter 8 with fraction multiplication and division. The last chapter (chapter 9) is about geometry. Students classify quadrilaterals and triangles, and learn about volume.

The part 5-A covers the four operations, place value and large numbers, problem solving, decimals and graphing.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

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User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

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Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Worktext 5-A	
Chapter 1	21 days
Chapter 2	12 days
Chapter 3	9 days
Chapter 4	18 days
Chapter 5	11 days
TOTAL	**71 days**

Worktext 5-B	
Chapter 6	20 days
Chapter 7	15 days
Chapter 8	20 days
Chapter 9	12 days
TOTAL	**67 days**

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A	88	10	178	78	2.28	11.4
5-B	92	10	188	82	2.29	11.5
Grade 5 total	180	20	366	160	2.29	11.4

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A			178			
5-B			188			
Grade 5 total			366			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

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In general, 1st-2nd graders might spend 25-40 minutes a day on maths. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

Working space, the usage of additional paper and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1000). Typically, I have intended that such exercises to be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

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Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

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Chapter 6: Decimals, Part 2
Introduction

This chapter focuses on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying and dividing decimals by powers of ten, presented with the help of place value charts. This is familiar to students from chapter 2, where they multiplied and divided whole numbers by powers of ten. The number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten.

As a shortcut, we can move the decimal point. However, the movement of the decimal point is an "illusion"—that is what seems to happen—but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value chart.

Next, we study how to multiply decimals by decimals. The common rule (or shortcut) for it says to multiply the numbers without the decimal points, and then add the decimal point to the product (answer) so that it has as many decimal digits as the factors have in total. We justify this rule using the recently learned technique for dividing decimals by powers of ten. Students are also encouraged to use estimation in decimal multiplications, and they solve problems connected to real life.

Then students learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0.4×1.2, as repeated addition. Instead, they are viewed as scaling—shrinking or enlarging—the number or quantity by a scaling factor. So, 0.4×1.2 is thought of as scaling 1.2 by 0.4, or as four-tenths *of* 1.2. You may recognise this as the same as 40% of 1.2.

Next, we go on to decimal divisions that can be done with mental maths. Students divide decimals by whole numbers (such as $0.8 \div 4$ or $0.45 \div 4$) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. $0.9 \div 0.3$ or $0.072 \div 0.008$).

In the lesson *More Division with Decimals*, we revise long division with decimals, when the divisor is a whole number.

Then, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekalitres to hectolitres. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimetre is a hundredth part of a metre, since the prefix "centi" means 1/100. Knowing that, gives us a means of converting between centimetres and metres.

The second lesson deals with more metric units, even those not commonly used, such as dekalitres and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Next, we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor).

After learning that, students practise measurement conversions within the customary system and do some generic problem solving with decimals.

Recall that not all students need all the exercises; use your judgement. Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

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Pacing Suggestion for Chapter 6

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Multiply and Divide by Powers of Ten, Part 1	13	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 2	16	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 3 (optional)	19	(*2 pages*)	(1 day)	
Multiply Decimals by Decimals 1	21	*2 pages*	1 day	
Multiply Decimals by Decimals 2	23	*3 pages*	1 day	
Multiplication as Scaling	26	*4 pages*	2 days	
Decimal Multiplication — More Practice	30	*2 pages*	1 day	
Dividing Decimals—Mental Maths	32	*3 pages*	1 day	
More Division with Decimals	35	*3 pages*	1 day	
The Metric System, Part 1	38	*4 pages*	2 days	
The Metric System, Part 2	42	*3 pages*	1 day	
Divide Decimals by Decimals 1	45	*3 pages*	1 day	
Divide Decimals by Decimals 2	48	*4-5 pages*	2 days	
Problem Solving	53	*4 pages*	2 days	
Mixed Revision Chapter 6	57	*2 pages*	1 day	
Chapter 6 Revision	59	*5 pages*	2 days	
Chapter 6 Test (optional)				
TOTALS		*48 pages*	20 days	
with optional content		(*51 pages*)	(21 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch6

SCAN ME

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Multiply and Divide by Powers of Ten 1

Remember? The number system we use is based on number 10. Therefore, each place value unit is always ten times the previous unit: 10 ones makes a ten, 10 tens makes a hundred, 10 hundreds makes a thousand. Because of this, when a number is multiplied by ten, the digits of the number essentially *move* in the place value chart!

Example 1. When 215 is multiplied by 10, each of its digits moves one slot to the left in the place value chart.

- The "2" in the hundreds place, signifying 200, becomes 2000.
- The "1" in the tens place, signifying 10, becomes 100.
- The "5" in the ones place (signifying 5) becomes 50.

Th	H	T	O		t	h	th
	2	**1**	**5**	**.**			

becomes

Th	H	T	O		t	h	th
2	**1**	**5**	**0**	**.**			

It works **the same way with decimals:** each place value unit is ten times the previous unit.

Example 2. 10 hundredths makes a tenth (or 10 × 0.01 = 0.1).

Using the place value chart, the digit one (signifying one hundredth) *moves* in the chart one slot to the left.

What if 0.01 was multiplied by 100?

10 × 0.01 = 0.1

Th	H	T	O		t	h	th
				.	1 ←	**1**	

Example 3. Since 10 × 0.01 = 0.1, it follows that 10 times *seven* hundredths equals seven tenths. The digit 7 moves in the place value chart one step to the left.

What if seven hundredths was multiplied by 100? By 1000?

What if there were other digits?

10 × 0.07 = 0.7

Th	H	T	O		t	h	th
				.	7 ←	**7**	

1. **a.** Using this technique, what happens to 7 thousandths when it is multiplied by 100? Explain, using the place value chart.

Th	H	T	O		t	h	th

b. What happens to 0.35 when it is multiplied by 1000? Explain.

Th	H	T	O		t	h	th

When you multiply a number by a power of ten (10, 100, 1000, etc.), each digit of the number *moves* in the place value chart as many steps as there are zeros in the power of ten.

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The Metric System, Part 1

The basic unit of LENGTH in the metric system is the **metre**. All the other units of length are formed by adding a prefix to the word “metre.”

For example, in “centimetre,” the prefix is *centi*, which signifies 1/100. This means that a centi-metre is 1/100 of a metre.

Or, in “hectometre,” the prefix is *hecto*, which signifies 100. So, a hecto-metre is 100 metres.

Notice that the conversion factor between each two neighbouring units is always **10**.

Units of Length in the Metric System

	kilometre	**km**	1000 metres
10	**hectometre**	**hm**	100 metres
10	**decametre**	**dam**	10 metres
10	**metre**	**m**	the basic unit
10	**decimetre**	**dm**	1/10 of a metre
10	**centimetre**	**cm**	1/100 of a metre
10	**millimetre**	**mm**	1/1000 of a metre

We can convert quantities with units with prefixes back to the basic unit by translating the prefix.

Example 1. Convert 26 cm into metres. Since *centi* signifies a hundredth, 26 centimetres is simply 26 *hundredths* of a metre, or 0.26 m.

Example 2. Since *kilo* signifies 1000, then 7 kilometres is 7 thousand metres, or 7000 m.

1. Convert these amounts to the basic unit, metres, by “translating” the prefixes.

a. 2 cm = *2/100 m* = *0.02 m*

6 dm = ________ m = ________ m

8 mm = ________ m = ________ m

b. 3 dam = _________ m

9 km = _________ m

2 hm = _________ m

c. 6 mm = _________ m

20 cm = _________ m

8 dm = _________ m

2. Now let’s look at metric units of weight, which are based on the basic unit **gram**. Convert these amounts to the basic unit, grams, by “translating” the prefixes.

a. 2 mg = *2/1000 g* = *0.002 g*

6 cg = _________ g = _________ g

8 dg = _________ g = _________ g

b. 7 dg = _________ g

6 kg = _________ g

8 dag = _________ g

c. 2 cg = _________ g

15 kg = _________ g

80 mg = _________ g

Units of Weight in the Metric System

	kilogram	**kg**	1000 grams
10	**hectogram**	**hg**	100 grams
10	**dekagram**	**dag**	10 grams
10	**gram**	**g**	the basic unit
10	**decigram**	**dg**	1/10 of a gram
10	**centigram**	**cg**	1/100 of a gram
10	**milligram**	**mg**	1/1000 of a gram

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Divide Decimals by Decimals 2

How to divide decimals by decimals:

1. First think how many times the divisor goes into the dividend. If you cannot figure this out with mental maths, go to step 2.
2. Multiply both the dividend and the divisor by 10 repeatedly until <u>the divisor is a whole number</u>. Then divide using long division.

Example 1. On the right, the division 0.644 ÷ 0.023 gets transformed into 644 ÷ 23. Each line is a division problem. At each step, the divisor and the dividend are multiplied by 10 —yet each problem has the <u>same answer</u>, 28.

	0.644 ÷ 0.023		= 28
× 10	6.44 ÷ 0.23	× 10	= 28
× 10	64.4 ÷ 2.3	× 10	= 28
× 10	644 ÷ 23	× 10	= 28

The shortcut for multiplying by 10 is to **move the decimal point.** So, at each step, when the dividend and the divisor are multiplied by 10, the decimal point moves one step to the right. You could, of course, simplify this process by moving the decimal point in both numbers *three* steps in one go.

Example 2. 2 2.4 4 0 ÷ 0.0 0 7 → 22 440 ÷ 7

Move the decimal point three steps. We need to add a zero.

We get a whole-number divisor. This problem can now be solved with long division.

1. Continue the patterns, multiplying the dividend and divisor in each step by 10, so that the quotients (the answers) remain the same.

a. 0.6 ÷ 0.02 = ________

6 ÷ ________ = ________

60 ÷ ________ = ________

600 ÷ ________ = ________

b. 0.48 ÷ 0.006 = ________

________ ÷ ________ = ________

________ ÷ ________ = ________

________ ÷ ________ = ________

2. Transform the problems so that you get a whole-number divisor. Then, divide using long division.

a. 44.7 ÷ 0.05

b. 7.588 ÷ 0.007

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3. Transform the problems so that you get a whole-number divisor. Then, divide using long division.

a. 10.401 ÷ 0.3	**b.** 380.9 ÷ 0.13
c. 1436 ÷ 0.004	**d.** 24.44 ÷ 2.6

Why does it work?

Naturally, the divisor "fits into" the dividend a certain number of times (which may not be a whole number). If you then multiply both the divisor and the dividend by the same number, the divisor will still "fit into" the dividend the same amount of times as before.

Or, look at it this way:

Remember equivalent fractions? We can multiply both numbers in a fraction by the same number (any number), and that will not change the value of the fraction.

$$\frac{3}{4} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{8}$$

Similarly, we can multiply both numbers in a division problem by the same number (any number), and that will not change the value of the division problem (the quotient).

$$\frac{57.6}{0.6} \overset{\times 10}{\underset{\times 10}{=}} \frac{576}{6} = 96$$

$$\frac{0.3402}{0.007} \overset{\times 10}{\underset{\times 10}{=}} \frac{3.402}{0.07} \overset{\times 10}{\underset{\times 10}{=}} \frac{34.02}{0.7} \overset{\times 10}{\underset{\times 10}{=}} \frac{340.2}{7} = 48.6$$

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Chapter 7: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade.

This chapter starts out with a lesson revising mixed numbers, and then with lessons on various ways to add and subtract mixed numbers. These are meant partially to revise and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as (1/5) − (4/5) = −3/5, and is not intended to become a "stumbling block." Simply skip it if necessary.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions (with different denominators). To that end, students learn how to convert fractions into other equivalent fractions. These lessons first use a visual model of splitting pie pieces further, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models, and then gradually advance toward the abstract. Several lessons are devoted to understanding and practising the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* revises some mental maths methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions.

Pacing Suggestion for Chapter 7

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 7	**page**	**span**	**suggested pacing**	**your pacing**
Fraction Terminology	65			
Revision: Mixed Numbers	66	*3 pages*	1 day	
Adding Mixed Numbers	69	*3 pages*	1 day	
Subtracting Mixed Numbers 1	72	*4 pages*	2 days	
Subtracting Mixed Numbers 2 (optional)	76	*(2 pages)*	(1 day)	
Equivalent Fractions 1	78	*3 pages*	1 day	
Equivalent Fractions 2	81	*2 pages*	1 day	
Adding and Subtracting Unlike Fractions	83	*3 pages*	1 day	
Finding the (Least) Common Denominator	86	*3 pages*	1 day	
Add and Subtract: More Practice	89	*3 pages*	1 day	
Adding and Subtracting Mixed Numbers	92	*3 pages*	1 day	
Comparing Fractions	95	*5 pages*	2 days	
Word Problems	100	*2 pages*	1 day	

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The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Mixed Revision Chapter 7	102	*3 pages*	1 day	
Chapter 7 Revision	105	*2.5 pages*	1 day	
Chapter 7 Test (optional)				
TOTALS		*37.5 pages*	15 days	
with optional content		*(39.5 pages)*	(16 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch7

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Fraction Terminology

As we study fraction operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can post it on your wall or even make your own fraction poster based on it. Some of the terms below you already know; some we will study in this chapter.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.

The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

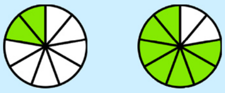
A mixed number has two parts: a whole-number part and a fractional part.

For example, in $2\frac{3}{7}$, the whole-number part is 2, and the fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

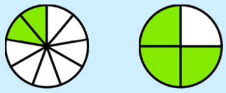
$2\frac{3}{7}$

Like fractions have the same denominator. They have the same kind of parts. It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts. It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

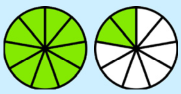


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

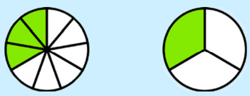
An improper fraction is more than 1 (more than a whole pie). Being a *fraction*, it is written as a fraction and *not* as a mixed number.

$\frac{2}{9}$ is a proper fraction.



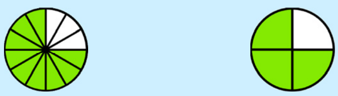
$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)



$\frac{9}{12}$ simplifies to $\frac{3}{4}$

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Subtracting Mixed Numbers 2

(This lesson is optional.)

Strategy 3 (optional): Subtract the whole numbers and the fractional parts separately. If you get a *negative* fraction, treat the "minus" as a subtraction symbol.

Example 1. $6\frac{2}{10} - 2\frac{5}{10} = ?$

- Subtract the whole numbers: $6 - 2 = \mathbf{4}$;
- Subtract the fractions: $\frac{2}{10} - \frac{5}{10} = \mathbf{-\frac{3}{10}}$.
- Combine the two results: $\mathbf{4} \mathbf{- \frac{3}{10}} = \mathbf{3\frac{7}{10}}$.

Example 2. $8\frac{1}{7} - 5\frac{6}{7} = ?$

- Subtract the whole numbers: $8 - 5 = \mathbf{3}$;
- Subtract the fractions: $\frac{1}{7} - \frac{6}{7} = \mathbf{-\frac{5}{7}}$.
- Combine the two results: $\mathbf{3} \mathbf{- \frac{5}{7}} = \mathbf{2\frac{2}{7}}$.

1. Subtract using any strategy. Remember to check that your answer is reasonable. If you subtract 5-and-something − 1-and-something, your answer cannot be 2-and-something.

a. $5\frac{3}{8} - 1\frac{7}{8} =$	**b.** $9\frac{2}{15} - 5\frac{8}{15} =$
c. $7\frac{11}{30} - 4\frac{9}{30} =$	**d.** $16\frac{5}{12} - 4\frac{11}{12} =$

2. You have 3 ¾ kg of ground beef. Your neighbour buys ¾ kg of it and you use ¾ kg to make meatballs. How much beef do you have left?

3. Subtract.

a. $5\frac{1}{11} - 3\frac{2}{11}$	**b.** $6\frac{6}{7} - 1\frac{5}{7}$	**c.** $6\frac{2}{15} - 1\frac{9}{15}$

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4. Find the missing minuend or subtrahend.

a. $\_\_\_ - 2\frac{1}{5} = 3\frac{2}{5}$	**b.** $\_\_\_ - 2\frac{6}{12} = 3\frac{5}{12}$	**c.** $7\frac{8}{9} - \_\_\_ = 4\frac{1}{9}$

5. Add and subtract.

a. $2\frac{1}{4} + 5\frac{3}{4} - 3\frac{2}{4} =$	**b.** $4\frac{5}{6} + 6\frac{3}{6} - 1\frac{4}{6} =$
c. $9\frac{3}{8} + 2\frac{7}{8} - 3\frac{6}{8} =$	**d.** $7\frac{7}{12} + 3\frac{11}{12} - 1\frac{2}{12} =$

6. Here are some more subtraction problems for additional practice, if your teacher assigns them. Colour the answer squares as given. Use extra paper to calculate.

a. (yellow) $5\frac{2}{9} - 2\frac{7}{9}$

b. (blue) $7\frac{8}{15} - 4\frac{11}{15}$

c. (blue) $5\frac{6}{11} - 3\frac{2}{11}$

d. (yellow) $4\frac{1}{9} - 2\frac{3}{9}$

e. (green) $17\frac{2}{9} - 4\frac{5}{9}$

f. (green) $5\frac{1}{11} - 3\frac{9}{11}$

g. (green) $10\frac{1}{12} - 4\frac{7}{12}$

h. (yellow) $4\frac{3}{10} - 2\frac{3}{10}$

i. (green) $5\frac{1}{10} - 3\frac{9}{10}$

j. (yellow) $8\frac{1}{8} - 2\frac{5}{8}$

k. (blue) $7\frac{1}{11} - 3\frac{5}{11}$

l. (blue) $9\frac{7}{8} - 3\frac{1}{8}$

m. (yellow) $15\frac{3}{12} - 10\frac{4}{12}$

$4\frac{11}{12}$	$1\frac{2}{10}$	$12\frac{6}{9}$	$2\frac{4}{9}$
$3\frac{7}{11}$	2		$2\frac{4}{11}$
$2\frac{12}{15}$			$6\frac{6}{8}$
$1\frac{7}{9}$	$1\frac{3}{11}$	$5\frac{6}{12}$	$5\frac{4}{8}$

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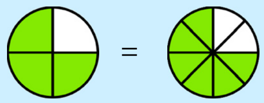
Equivalent Fractions 1



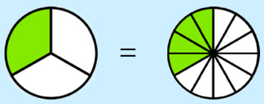
× 2

$\frac{3}{4} = \frac{6}{8}$

× 2

These two fractions are **equivalent fractions** because they picture the same, or equal, amounts. You could say that you get to "eat" the same amount of "pie" either way.

In the second picture, **each slice** has been **split or cut into two pieces**. The arrows show into how many new pieces each piece was split.



× 4

$\frac{1}{3} = \frac{4}{12}$

× 4

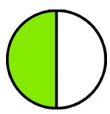
Each slice has been split into four.

BEFORE: 1 coloured piece, 3 total.
AFTER: 4 coloured pieces, 12 total.

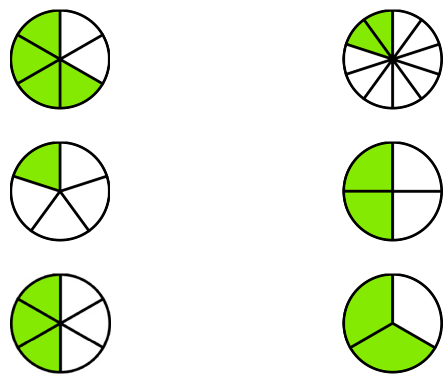
Notice that we get *four* times as many coloured pieces and *four* times as many total pieces. This means that both the numerator and the denominator get multiplied by 4.

When all of the pieces are split the same way, both the number of coloured pieces (the numerator) and the total number of pieces (the denominator) get multiplied by the same number.

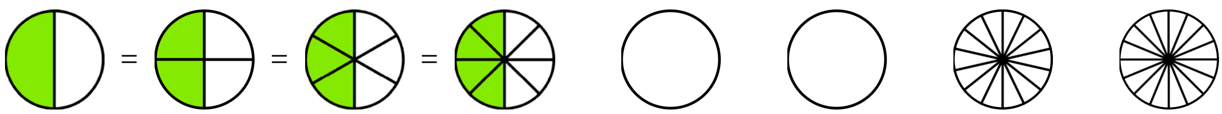
1. Connect the pictures that show equivalent fractions. Write the name of each fraction beside its picture.



$\frac{1}{2}$



2. Make a chain of equivalent fractions. Notice the patterns!



$\frac{1}{2}$ = = = = $\frac{5}{\ }$ = $\frac{6}{\ }$ = =

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Chapter 8: Fractions: Multiply and Divide
Introduction

This is another chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. If you feel that your student(s) would benefit from taking a break from fractions, you could have them study chapter 9 (geometry) in between chapters 7 and 8.

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 7, it should be relatively simple for students to understand. We also use the same visual model, just backwards: this time the pie pieces are joined together instead of split apart.

Next we study multiplying a fraction and a whole number. The lesson shows how, for example, 3 × (4/5) can be seen as three copies of 4/5 — as repeated addition. In this case, all that is needed is to find the number of fifths (number of slices), and that is simply 3 × 4.

We also delve into the idea of interpreting a fraction times a whole number as a fractional part of a quantity. For example, (2/3) × 18 is seen as two-thirds of 18 (say 18 km or $18). In this sense, the word "of" is as if it "translates" into the multiplication symbol.

The next lesson continues to build on this idea, explaining the multiplication of a fraction by a fraction as taking a certain part of a fraction. The lesson also shows the usual shortcut for the multiplication of fractions.

Then, we study the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies, and also because it makes fraction multiplication easier. I have also included explanations of *why* we are allowed to simplify before multiplying, so that students can become familiar with mathematical reasoning (actually, proofs).

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognising that 5/3 is the same as 5 ÷ 3. This gives us a means of dividing whole numbers in such a manner that the answer has a fractional part (for example, 20 ÷ 6 = 3 2/6).

The next case is sharing divisions—divisions that can be interpreted as equal sharing. For example, if 4/5 of a pie is shared equally between two people, how much does each person get? In particular, we look at dividing a unit fraction by a whole number (e.g. (1/4) ÷ 3) in this context of equal sharing. Students work with visual models, and via their work, find a shortcut for this type of division.

The following lesson then focuses on "measurement divisions", where we think how many times the divisor "fits into" the dividend. Again, visual models help a lot. The focus is on dividing a whole number by a unit fraction (e.g. 3 ÷ (1/4)).

The last lesson, on the shortcut for fraction division, is optional. It reveals the common rule for fraction division: each division is actually changed into a *multiplication* by the reciprocal of the divisor. In 5th grade, students are not required to master fraction division in all cases, and that is why this is an optional lesson. This rule is studied in 6th grade in detail.

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Pacing Suggestion for Chapter 8

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.
* These lessons exceed the Common Core Standards (CCS) for 5th grade.

The Lessons in Chapter 8	page	span	suggested pacing	your pacing
* Simplifying Fractions 1	111	*3 pages*	1 day	
* Simplifying Fractions 2	114	*3 pages*	1 day	
Multiply Fractions and Whole Numbers 1	117	*2 pages*	1 day	
Multiply Fractions and Whole Numbers 2	119	*2 pages*	1 day	
Multiply Fractions by Fractions 1	121	*3 pages*	1 day	
Multiply Fractions by Fractions 2	124	*2 pages*	1 day	
Fraction Multiplication and Area	126	*6 pages*	2 days	
* Simplifying Before Multiplying	133	*3 pages*	1 day	
Multiplying Mixed Numbers	136	*3 pages*	1 day	
Multiplication as Scaling/Resizing	139	*3 pages*	2 days	
Fractions Are Divisions	142	*4 pages*	2 days	
Dividing Fractions: Sharing Divisions	146	*3 pages*	1 day	
Dividing Fractions: Fitting the Divisor	149	*3 pages*	1 day	
Dividing Fractions: Summary	152	*2 pages*	1 day	
* Dividing Fractions: The Shortcut (optional)	154	*(3 pages)*	(1 day)	
Mixed Revision Chapter 8	157	*3 pages*	1 day	
Chapter 8 Revision	160	*4 pages*	2 days	
Chapter 8 Test (optional)				
TOTALS		*49 pages*	20 days	
with optional content		*(52 pages)*	(21 days)	

Helpful Resources on the Internet

These resources match the topics in this chapter, and offer online practice, online games, and interactive illustrations of maths concepts. We heartily recommend you take a look. Many people love using these resources to supplement the bookwork, to illustrate a concept better, and for some fun. Enjoy!

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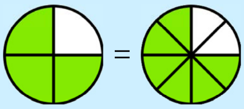
Simplifying Fractions 1

You have learned how to convert a fraction into an equivalent fraction:

Each slice is **split two ways.**

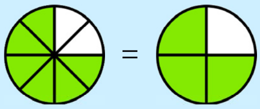


$$\underset{\times 2}{\overset{\times 2}{\frac{3}{4} = \frac{6}{8}}}$$

What happens if we *reverse* the process?

Then it is called **SIMPLIFYING** or **REDUCING** a fraction:

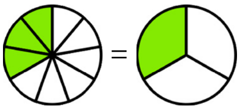
Every two slices are **joined together**.



$$\underset{\div 2}{\overset{\div 2}{\frac{6}{8} = \frac{3}{4}}}$$

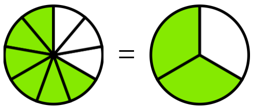
1. Simplify the following fractions, filling in the missing parts.

a. Every *three* slices are joined together.



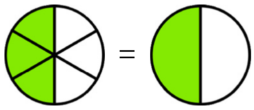
$$\underset{\div \square}{\overset{\div 3}{\frac{3}{9} = \frac{\square}{\square}}}$$

b. Every ________ slices are joined together.



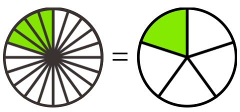
$$\underset{\div \square}{\overset{\div \square}{\frac{\square}{\square} = \frac{\square}{\square}}}$$

c. Every ________ slices are joined together.



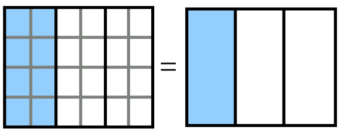
$$\underset{\div \square}{\overset{\div \square}{\frac{\square}{\square} = \frac{\square}{\square}}}$$

d. Every ________ slices were joined together.



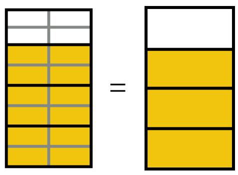
$$\underset{\div \square}{\overset{\div \square}{\frac{\square}{\square} = \frac{\square}{\square}}}$$

e. Every ________ parts were joined together.



$$\underset{\div \square}{\overset{\div \square}{\frac{\square}{\square} = \frac{\square}{\square}}}$$

f. Every ________ parts were joined together.



$$\underset{\div \square}{\overset{\div \square}{\frac{\square}{\square} = \frac{\square}{\square}}}$$

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Multiplication as Scaling/Resizing

You know that **scaling** means **expanding or shrinking something** by some factor.

We use **multiplication** to accomplish this. The number we multiply by is called the **scaling factor**.

Example 1. When a stick 40 pixels long is scaled to be 3/5 as long as it was, it will shrink!

→

We could write this type of a multiplication equation: (3/5) × = .

Using the length of 40 pixels, we write (6/10) × 40 px = 24 px or 0.6 × 40 px = 24 px.

Example 2. The multiplication (1 2/3) × 18 km means taking the distance of 18 km one and two-thirds times. We are scaling the quantity 18 km by the factor 1 2/3.

To calculate it, we can multiply in parts: take 1 × 18 km, and (2/3) × 18 km, and add those. Since two-thirds of 18 km is 12 km, then **(1 2/3) × 18 km** is **18 km + 12 km = 30 km.**

1. The stick and other quantities are being scaled—either expanded or shrunk. Find the quantity after scaling. Compare the problems in each box.

a.

½ × =

½ × 50 px = ________ px

1 ½ × =

1 ½ × 50 px = ________ px

b.

¼ × =

¼ × 40 px = ________ px

2 ¼ × =

2 ¼ × 40 px = ________ px

c.

⅝ × 400 km = ___________

2 ⅝ × 400 km = ___________

d.

⅗ × $600 = __________

3 ⅗ × $600 = __________

2. A 1200×800 photo (in pixels) is scaled by scaling factor *s*.

a. If you want the resulting photo to be slightly smaller than the original, what kind of number would you use for *s*?

b. If *s* = 2 ¾, calculate the dimensions of the resulting photo.

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3. Will the resulting stick be longer or shorter than the original — or equally long? You do not have to calculate anything. Compare.

a. $\frac{9}{8}$ × ——— is longer/shorter than ——— .	**b.** $\frac{3}{7}$ × ——— is longer/shorter than ——— .
c. $3\frac{2}{100}$ × ——— is longer/shorter than ——— .	**d.** $\frac{99}{100}$ × ——— is longer/shorter than ——— .

4. Let *s* be the scaling factor. For what kind of values of *s* will *s* × \$500 be more than \$500? For what kind of values will it be less?

5. Write < , > , or = in the boxes. Fill in a number on the empty lines.

A quantity (or a number) is scaled by scaling factor *s*.

When *s* ☐ ___ , the resulting quantity is more than the original.

When *s* ☐ ___ , the resulting quantity is less than the original.

When *s* ☐ ___ , the resulting quantity is equal to the original.

6. Scaling is also the concept we use when calculating prices. Find the total cost. Use either fractions or decimals, depending on what makes most sense.

a. Nuts cost \$17.50/kg. You buy 1 1/2 kg.

b. It costs \$350 to rent a room for 30 days. What does it cost for 12 days?

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A neat connection.

You have learned to use multiplication with equivalent fractions. →

$$\frac{3}{4} \overset{\times 5}{\underset{\times 5}{=}} \frac{15}{20}$$

We can write the same process this way: $\frac{3}{4} = \frac{5 \times 3}{5 \times 4} = \frac{15}{20}$

Notice: $\frac{5 \times 3}{5 \times 4}$ is the same as $\frac{5}{5} \times \frac{3}{4}$, isn't it? And $\frac{5}{5}$ is equal to 1.

Therefore, $\frac{5 \times 3}{5 \times 4}$ is actually the same as multiplying $\frac{3}{4}$ by 1.

So, we can make equivalent fractions by multiplying a given fraction by 1;
we just write 1 in the form of a fraction (such as 3/3 or 11/11).

7. Make equivalent fractions by multiplying the given fraction by different forms of the number 1.

a. Multiply by $\frac{4}{4}$. $\frac{\square}{\square} \times \frac{2}{3} =$	**b.** Multiply by $\frac{3}{3}$. $\frac{\square}{\square} \times \frac{5}{9} =$	**d.** Multiply by $\frac{7}{7}$. $\frac{\square}{\square} \times \frac{11}{12} =$

8. Kathy multiplied $\frac{2}{7}$ by $\frac{10}{10}$, and said it didn't really change anything. Heather chimed in and said, "No, it's 10 times bigger now."

a. Is Heather correct? Explain why or why not.

b. Which number is 10 times bigger than 2/7?

9. Is the result of multiplication more, less, or equal to the original number? You do not have to calculate anything. Compare, writing < , > , or = in the box.

a. $\frac{9}{10} \times 16$ $\square$ 16	**b.** $5\frac{7}{9} \times 31$ $\square$ 31	**c.** $\frac{6}{6} \times 5$ $\square$ 5
d. $\frac{20}{20} \times 88$ $\square$ 88	**e.** $\frac{4}{5} \times \frac{2}{3}$ $\square$ $\frac{2}{3}$	**f.** $\frac{11}{4} \times 164$ $\square$ 164
g. $\frac{7}{5} \times \frac{4}{4}$ $\square$ $\frac{7}{5}$	**h.** 2.61×7 $\square$ 7	**i.** 0.918×431 $\square$ 431

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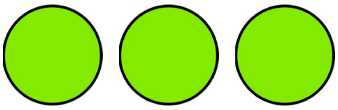
Fractions Are Divisions

1. You want to share *two* pies evenly between *three* people.
   How will that work? What part (fraction) of one pie will each person get?
   Use drawings to explore the situation.

2. Continue exploring these types of uneven divisions. Fill in, the best you can. If you feel confused, don't worry! We will look at this concept in more detail on the next page.

a. Divide 3 pies equally among four people. In other words, solve 3 ÷ 4.

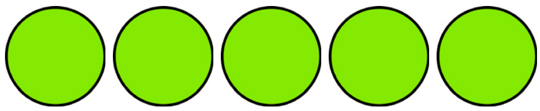


Each will get $\frac{}{}$ of a pie.

b. Divide 3 protein bars equally among five people. In other words, solve 3 ÷ 5.

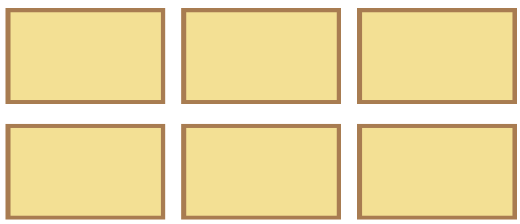
Each will get $\frac{}{}$ of a bar.

c. Divide 5 pies equally among six people. In other words, solve 5 ÷ 6.



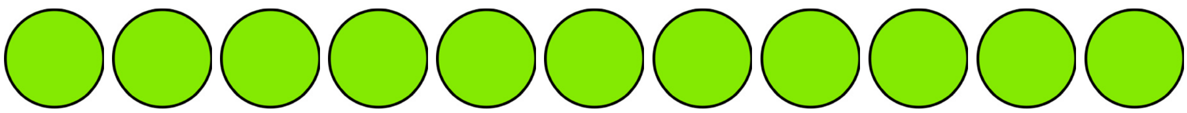
Each person will get $\frac{}{}$ of a pie.

d. Divide 6 protein bars equally among four people. In other words, solve 6 ÷ 4.



Each will get ________ bars.

e. Divide 11 pies equally among eight people. In other words, solve 11 ÷ 8.



Each person will get ________ pies.

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Chapter 9: Geometry
Introduction

The focus of this chapter is on two topics: classifying two-dimensional shapes, and volume.

The chapter starts out with a lesson that revises the topic of angles from fourth grade. The next lesson (Polygons) covers the concept of a polygon and the names of several common ones. Students classify figures into polygons and non-polygons, and also into regular polygons versus non-regular polygons.

The next topic is classifying quadrilaterals. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the "children" (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of a rhombus).

A possible confusion point is the definition of a trapezium. There exist two possible definitions:

- (Exclusive definition:) A trapezium has exactly one pair of parallel sides.
- (Inclusive definition:) A trapezium has at least one pair of parallel sides.

Both definitions are legitimate, but lead to different analysis when classifying quadrilaterals. Under the exclusive definition, a parallelogram is not a trapezium, but under the inclusive definition, it is. Most college-bound textbooks favour the *inclusive* definition, and that is what is used in this text, also.

Then we study the classification of triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles.

The second focus topic of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. They find the volume of right rectangular prisms by "packing" them with unit cubes and by using formulas. They recognise volume as additive and solve both geometric and real-word problems involving volume.

The chapter includes three optional lessons listed in the end: area and perimeter problems, star polygons, and circles. Use them as time allows. The lesson on area and perimeter can be important for those students who tend to forget these concepts. The lesson on star polygons is intended as a fun artistic topic. The lesson on circles involves the usage of a compass, which may be hard for some children at this age. Those who can master it will probably find the exercises involving multiple circles fascinating.

Note: Any problem marked with "  " means the exercise should be done in a notebook or on blank paper.

Pacing Suggestion for Chapter 9

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 9	page	span	suggested pacing	your pacing
Geometry Vocabulary Reference Sheet	167			
Revision: Angles	178	*3-4 pages*	1 day	
Polygons	172	*3 pages*	1 day	
Classifying Quadrilaterals 1	175	*3 pages*	1 day	
Classifying Quadrilaterals 2	178	*3 pages*	1 day	
Classifying Quadrilaterals 3 (optional)	181	*(2 pages)*	(1 day)	

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The Lessons in Chapter 9	page	span	suggested pacing	your pacing
Classifying Triangles 1	183	*3 pages*	1 day	
Classifying Triangles 2	186	*2 pages*	1 day	
Volume	188	*5 pages*	2 days	
Volume of Rectangular Prisms	193	*3 pages*	1 day	
Volume is Additive	196	*3 pages*	1 day	
* Area and Perimeter Problems (optional)	199	*(5 pages)*	(2 days)	
* Star Polygons (optional)	203	*(2 pages)*	(1 day)	
Mixed Revision Chapter 9	205	*3 pages*	1 day	
Chapter 9 Revision..	208	*3 pages*	1 day	
Chapter 9 Test (optional)				
TOTALS		*34 pages*	12 days	
with optional content		*(44 pages)*	(16 days)	

* These lessons exceed the Common Core Standards (CCS) for 5th grade.

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch9

SCAN ME

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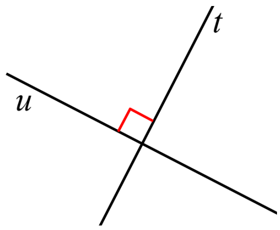
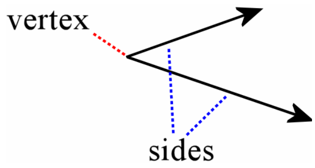
Geometry Vocabulary Reference Sheet

I encourage you to draw pictures to illustrate the terms, or even make your own geometry notebook!

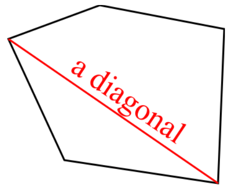
Two lines are **perpendicular** if they form a right angle.



An **angle** consists of two rays that start at the same point, called vertex. The two rays form the sides of the angle.



- A **polygon** is a flat, two-dimensional figure that consists of line segments, and is closed.
- A **regular polygon** is one with congruent sides and angles.
- A **vertex** is a “corner” of a polygon.
- A **diagonal** is a line segment drawn from one vertex of a polygon to another.



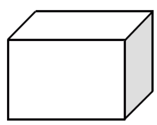
- A **quadrilateral** – a polygon with *four* sides
- A **pentagon** – a polygon with *five* sides.
- A **hexagon** – a polygon with *six* sides.
- A **heptagon** – a polygon with *seven* sides.
- An **octagon** – a polygon with *eight* sides.

- A **right triangle** is a triangle with one right angle.
- An **obtuse triangle** is a triangle with one obtuse angle.
- An **acute triangle** is a triangle with all three angles acute.

- An **equilateral triangle** is a triangle with three congruent sides.
- An **isosceles triangle** is a triangle with two congruent sides.
- A **scalene triangle** is a triangle where none of the sides are congruent.

- A **trapezium** is a quadrilateral with at least one pair of parallel sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- A **rhombus** is a parallelogram with four congruent sides.
- A **kite** is a quadrilateral that has two pairs of congruent sides, and the congruent sides are adjacent (neighbouring each other).
- A **rectangle** is a quadrilateral with four right angles.
- A **square** is a rectangle with four congruent sides.
- A **scalene quadrilateral** has no congruent sides.

- A **rectangular prism** is a box-shaped solid (three-dimensional shape) with edges that meet at right angles.



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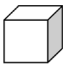
Volume

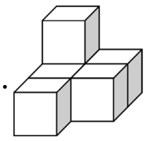
The **volume** of an object has to do with how much SPACE it takes up or occupies.

You have measured the volume of liquids using measuring cups that use millilitres.
If we need to know the volume of a big object, such as a room, we cannot pour water into it to measure it with measuring cups. Instead, we use cube-shaped units or **cubic units**, and we simply check or calculate how many cubic units fit into the object.

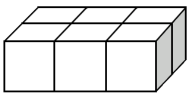
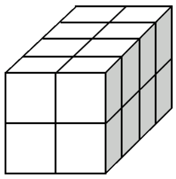
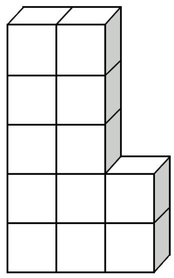
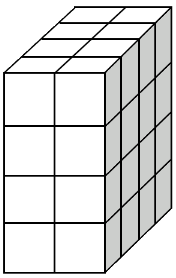
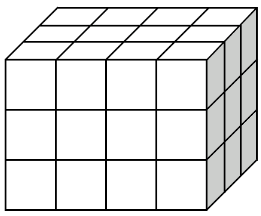
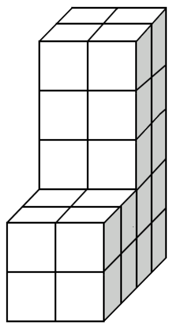
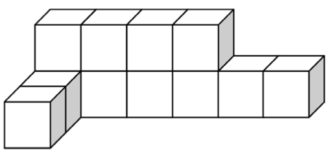
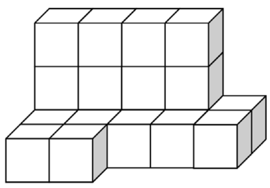
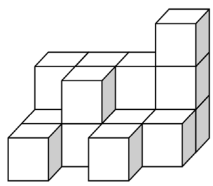


This little cube is **1 cubic unit.**

The volume of the figure on the right is six cubic units: V = 6 cubic units.
Notice that one cube is not visible.



1. Find the volume of these figures in cubic units. “V” means volume.

 **a.** V = ________ cubic units	 **b.** V = ________ cubic units	 **c.** V = ________ cubic units
 **d.** V = ________ cubic units	 **e.** V = ________ cubic units	 **f.** V = ________ cubic units
 **g.** V = ________ cubic units	 **h.** V = ________ cubic units	 **i.** V = ________ cubic units

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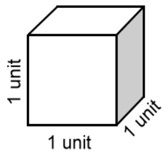
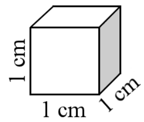


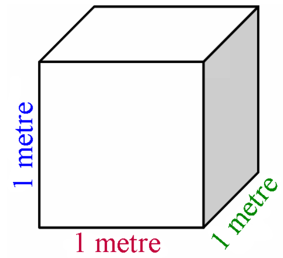
If each edge of the cube is 1 unit, its volume is **1 cubic unit.**

V = 1 cubic unit or 1 unit^3



If each edge of the cube is 1 cm, its volume is **1 cubic centimetre**.

V = 1 cm^3

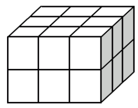


If each edge of the cube is 1 metre, its volume is **1 cubic metre.**

V = 1 m^3

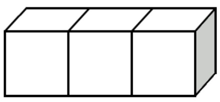
In general, if the edges of the cube are in certain units (such as centimetres, or metres), then the volume will be in corresponding *cubic* units.

If no unit is given for the edge lengths, we use the word “unit” for the lengths of the edges, and “cubic unit” for the volume. This “box” has a volume of 18 cubic units.



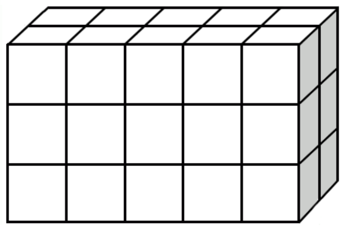
2. Find the total volume of each figure when the edge length of the little cube is given. Remember to include the unit!

The edge of each cube is 1 unit.



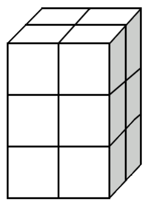
a. V = 3 unit^3

The edge of each cube is 1 m.



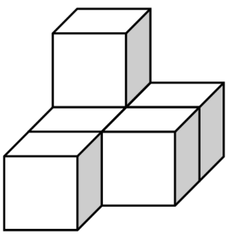
b. V = ______________

The edge of each cube is 1 cm.



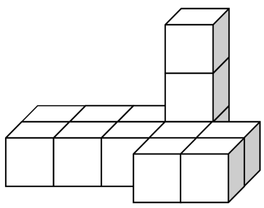
c. V = ______________

The edge of each cube is 1 m.



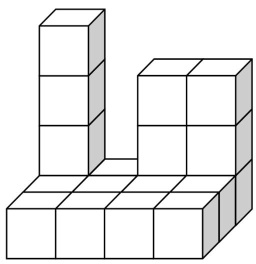
d. V = ______________

The edge of each cube is 1 cm.



e. V = ______________

The edge of each cube is 1 m.



f. V = ______________

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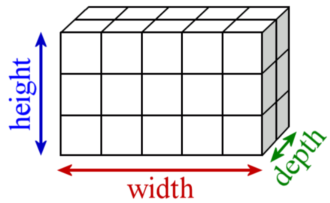
This figure is called a **rectangular prism.** It is also called *a cuboid*.
It is simply a box with sides that meet at right angles.

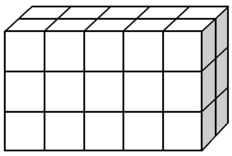


Many people call the **three dimensions** that we measure “length,” “width,” and “height.” Here we will use “width,” “depth,” and “height.”

The **width** will be the dimension that runs left to right.
The **depth** will be the dimension that points away from you—into the paper, so to speak.
The **height** will be the dimension pointing “up” in the figure.

A way to find the volume of a rectangular prism by calculating

1) Can you figure out a way to find the number of cubes in the *bottom* layer of this rectangular prism *without* counting?

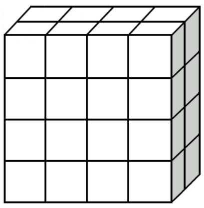
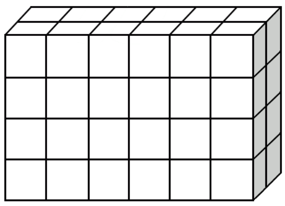
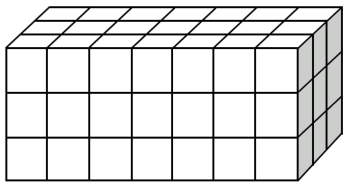
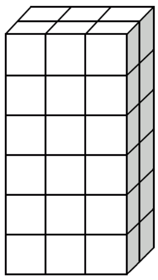


You can multiply 5 × 2 = 10, which means multiplying the *width* and the *depth*. The bottom layer has 10 cubic units.

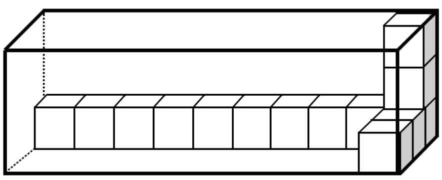
2) After that, there is a way to easily find the *total* number of cubes in the rectangular prism (its volume). Can you figure that out?

Take the number of cubes in the bottom layer, and **multiply that by how many layers there are** (the *height*). There are 10 cubes in the bottom layer, and 3 layers.
We get 10 × 3 = 30 cubic units.

3. Find the volume of these rectangular prisms by finding the amount of cubic units in the bottom layer and multiplying that by the height (how many layers there are).

	a.	**b.**	**c.**	**d.**
				
Cubes in the bottom layer	*8*			
Height	*4*			
Volume	*32*			

4. If each little cube is 1 cubic centimetre, what is the total volume of the outer box?



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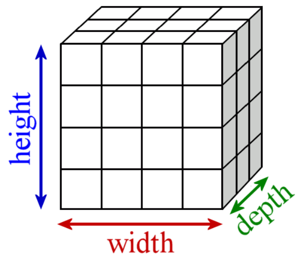
Notice what we did in these two steps:

(1) We multiplied the width and the depth to find the number of cubes in the bottom layer. **Multiplying the width and the depth** also gives us **the area of the bottom** (A_b)! For example, the bottom area of this cuboid is 4 × 3 = 12 square units.



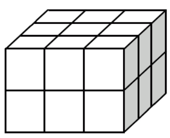
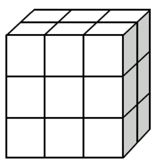
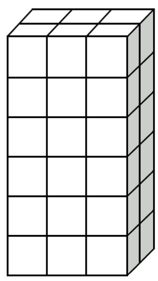
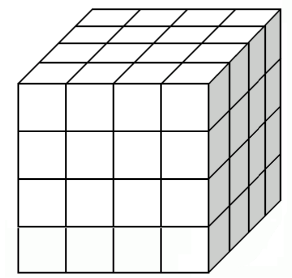
(2) We multiplied what we got from step 1 by height.

We ended up multiplying the bottom area by the height.
Or, looking at it in another way, we multiplied the width, the depth, and the height.

From that we get two **formulas** for the volume of a rectangular prism:

1. $V = w \times d \times h$ (volume is width × depth × height)
2. $V = A_b \times h$ (volume is area of the bottom × height)

5. Write the width, height, and depth of these rectangular prisms. Lastly, multiply those three dimensions to find the volume.

	a.	**b.**	**c.**	**d.**
				
Width:	*3*			
Depth:	*3*			
Height:	*2*			
Volume:				

6. Find the volume of the rectangular prisms above *if* their top layer was removed. Use cubic units. Use the formula $V = w \times d \times h$.

a. V = *3* × *3* × *1* = _______ cubic units

b. V = _____ × _____ × _____ = _______ cubic units

c. V = _____ × _____ × _____ = _______ cubic units

d. V = _____ × _____ × _____ = _______ cubic units

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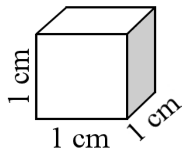
Now we can explain where the little raised "3" (the exponent) in cubic units comes from.

To find the volume of this cube, we multiply its width, the height, and the depth. This means multiplying 1 cm × 1 cm × 1 cm. Not only do we multiply number 1 by itself three times—we also multiply the *unit centimetre* (cm) by itself three times. The little "3" in cm^3 shows that.



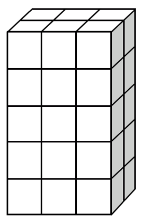
7. **a.** Sketch a rectangular prism with a volume of 4 × 2 × 6 cubic units.

b. Sketch a rectangular prism with a volume of 3 × 3 × 3 cubic units.

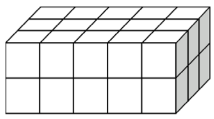
c. Sketch a rectangular prism with a volume of 2 × 5 × 4 cubic units.

8. Chris and Mia drew these rectangular prisms to match the expression 5 × 3 × 2. Who is right?

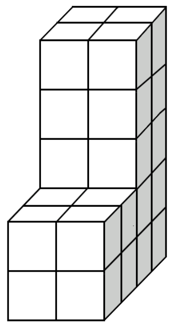
Chris:



Mia:



9. To calculate the volume of this kind of figure, think of it as consisting of *two* rectangular prisms. We calculate the volume of each separately, and then add. Which expression below matches the volume of this figure?



a. 2 × 3 × 2 + 2 × 2 × 3

b. 2 × 2 × 2 + 2 × 2 × 3

c. 2 × 2 × 2 + 2 × 2 × 5

Sample worksheet from https://www.mathmammoth.com

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Volume of Rectangular Prisms

Study the two formulas for the volume of a rectangular prism:

1. $\mathbf{V} = \boldsymbol{w} \times \boldsymbol{d} \times \boldsymbol{h}$ (volume is width × depth × height)
   Some people use width, <u>length</u>, and height instead.

2. $\mathbf{V} = \mathbf{A_b} \times \boldsymbol{h}$ (volume is area of the bottom × height)

The width, depth, and height need to be in the <u>same</u> kind of unit of length (such as metres). The volume will then be in corresponding cubic units (such as cubic metres).

Example 1. A room measures 3 m by 2 m, and it is 3 m high. What is the volume of the room? What is the area of the room?

To find the area, we simply multiply the two given dimensions: A = 3 m × 2 m = 6 m^2.
To find the volume, we can multiply the area by the height: V = 6 m^2 × 3 m = 18 m^3.

1. **a.** Find the volume of a box that is 12 cm high, 5 cm wide, and 10 cm deep. Include the units! V = <u>12 cm</u> × ____________ × ____________ = ____________

   b. Find the area and volume of a room that is 5 m × 4 m, and 3 m high. Include the units!

   A = ____________ × ____________ = ____________

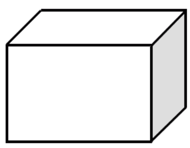
   V = ____________ × ____________ × ____________ = ____________

2. Find the volume of a box that is

   a. 20 cm wide, 30 cm deep, and half a metre high.
   Note: you will need to convert the last measurement into centimetres before calculating the volume.

   b. 16 square cm on the bottom, and 6 cm tall.

3. The volume of this box is 30 cm^3.
   What is its depth?



3 cm

5 cm

4. *Optional*. Measure the width, height, and depth of a dresser and/or a fridge.
   Find out its volume.

Sample worksheet from
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