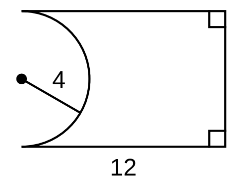
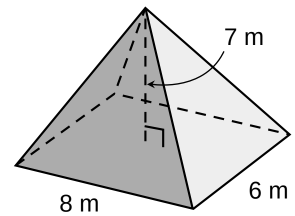
27. (61,63) Tell whether the following numbers are rational numbers or irrational numbers:
(a) $2\sqrt{3}$ (b) $7.\overline{231}$ (c) -14.7 (d) $\sqrt{289}$

28. (66) Simplify: $3\sqrt{18} - 7\sqrt{8} + 3\sqrt{50} + \sqrt{32}$

29. (3) Find the perimeter of this figure. Dimensions are in centimeters.



30. (72) Find the volume of the right rectangular pyramid.



LESSON 73 Factoring the Difference of Two Squares • Probability Without Replacement

73.A factoring the difference of two squares

Since each of the terms in the following binomials is a perfect square,

$$x^2 - y^2 \qquad 4p^2 - 25 \qquad m^2 - 16$$

these binomials are sometimes called **the difference of two squares.** They can be generated by multiplying the sum and difference of two monomials.

$$\begin{array}{lll}
\begin{array}{r} x + y \\ \underline{x - y} \\ x^2 + xy \\ \underline{\quad - xy - y^2} \\ x^2 \qquad - y^2 \end{array} &
\begin{array}{r} 2p + 5 \\ \underline{2p - 5} \\ 4p^2 + 10p \\ \underline{\quad - 10p - 25} \\ 4p^2 \qquad - 25 \end{array} &
\begin{array}{r} m + 4 \\ \underline{m - 4} \\ m^2 + 4m \\ \underline{\quad - 4m - 16} \\ m^2 \qquad - 16 \end{array}
\end{array}$$

We note in each case that the middle term is eliminated because the numerical coefficients of the addends that would form the middle term have the same absolute value but are opposite in sign.

If we are asked to factor a binomial that is the difference of two squares, such as

$$9m^2 - 49$$

the problem is a problem in recognition. There is no procedure to follow. We recognize that each term of the binomial is a perfect square and that the binomial can be written as

$$(3m)^2 - (7)^2$$

Now from the pattern developed above, we can write

$$9m^2 - 49 = (3m + 7)(3m - 7)$$

In general, the difference of two squares, $F^2 - S^2$, can be factored as follows:

$$F^2 - S^2 = (F - S)(F + S)$$

example 73.1 Factor: $-4 + x^2$

solution **We recognize that both of the terms are perfect squares.** We begin by reversing the order of the terms and writing the squared terms as

$$x^2 - 4 = (x)^2 - (2)^2$$

and now we can write the factored form as

$$\mathbf{(x + 2)(x - 2)}$$

example 73.2 Factor: $49m^2 - a^2$

solution **We recognize that each of the terms is a perfect square** and that the binomial can be written as

$$(7m)^2 - (a)^2$$

The factored form of this binomial is

$$\mathbf{(7m + a)(7m - a)}$$

example 73.3 Factor: $-36a^2 + 25y^2$

solution **We recognize that both of the terms are perfect squares.** We begin by reversing the order of the terms and writing the squared terms as

$$25y^2 - 36a^2 = (5y)^2 - (6a)^2$$

Now we write the factored form as

$$\mathbf{(5y + 6a)(5y - 6a)}$$

example 73.4 Factor: $-36x^6y^4 + 49a^2$

solution **Again we recognize that both of the terms are perfect squares.** We begin by rearranging the order of the terms and then factoring by inspection.

$$\mathbf{\left(7a + 6x^3y^2\right)\left(7a - 6x^3y^2\right)}$$

73.B probability without replacement

When we make successive random selections of marbles from an urn, the probability of a certain outcome on the second draw is affected by whether or not the marble selected on the first draw is returned before the second draw is made.

example 73.5 An urn contains 3 black marbles and 5 white marbles. A marble is drawn at random and replaced. Then a second marble is randomly drawn. (a) What is the probability that both marbles are black? (b) If the first marble is not replaced before the second marble is drawn, what is the probability that both marbles are black?

solution (a) The probability of a black marble on the first draw is $\frac{3}{8}$. Since the first marble is replaced, the probability of a black marble on the second draw is also $\frac{3}{8}$. So

$$P(\text{both black}) = \frac{3}{8} \cdot \frac{3}{8} = \mathbf{\frac{9}{64}}$$

(b) The probability of a black marble on the second draw is not the same because the first marble was not replaced. If the first draw was black, then the probability of a black marble on the second draw is $\frac{2}{7}$ because only 2 black marbles and 7 marbles total remain. Thus, the probability of 2 black marbles when there is no replacement between draws is

$$P(\text{both black}) = \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56} = \mathbf{\frac{3}{28}}$$

example 73.6 An urn contains 4 red marbles and 7 blue marbles. Two marbles are drawn at random. What is the probability that the first is red and the second is blue if the marbles are drawn (a) with replacement? (b) without replacement?

solution (a) The probability of a red marble on the first draw is $\frac{4}{11}$. Since the first marble is replaced, the probability of a blue marble on the second draw is $\frac{7}{11}$. So

$$P(\text{red, then blue}) = \frac{4}{11} \cdot \frac{7}{11} = \mathbf{\frac{28}{121}}$$

(b) The probability of a blue marble on the second draw is not the same because the first marble was not replaced. If the first draw was red, then the probability of a blue marble on the second draw is $\frac{7}{10}$ because only 10 marbles remain. Thus, the probability of drawing one red marble, then one blue marble when there is no replacement between draws is

$$P(\text{red, then blue}) = \frac{4}{11} \cdot \frac{7}{10} = \frac{28}{110} = \mathbf{\frac{14}{55}}$$

practice

Factor these binomials:

a. $64x^2 - 81y^2$ **b.** $-25 + 100m^2$ **c.** $y^4x^2 - 169z^{10}$

d. An urn contains 4 purple marbles and 3 pink marbles. Two marbles are drawn at random. What is the probability that both marbles are purple if the marbles are drawn

(1) with replacement? (2) without replacement?

e. An urn contains 5 orange marbles and 6 blue marbles. Two marbles are drawn at random. What is the probability that the first marble is orange and the second marble is blue if the marbles are drawn

(1) with replacement? (2) without replacement?

problem set 73

1. *(73)* An urn contains 6 purple marbles and 4 pink marbles. A marble is drawn at random and not replaced. Then a second marble is drawn. What is the probability that both marbles are purple?

2. *(70)* An urn contains 2 orange marbles and 5 blue marbles. A marble is drawn at random and replaced. Then a second marble is drawn. What is the probability that the first marble is orange and the second marble is blue?

3. *(70)* A fair coin is tossed three times. What is the probability that the first two tosses come up tails and the third toss comes up heads?

4. *(70)* Two dice are rolled. What is the probability that the sum of the numbers rolled is

(a) 3? (b) a number less than 3?

5. *(58)* Rosemary saw 900 of them in all. If this number was 150 percent greater than the number she expected to see, how many did she expect to see? Draw a diagram as an aid in solving the problem.

6. *(52)* Hannibal noted that the average weight of the first 4 animals was 2000 pounds. The average weight of the next 96 animals was only 100 pounds. What was the average weight of all the animals?

Factor these binomials.

7. *(73)* $4p^2x^2 - k^2$ **8.** *(73)* $-4m^2 + 25p^2x^2$ **9.** *(73)* $-9x^2 + 4y^2$

10. *(73)* $9k^2a^2 - 49$ **11.** *(73)* $p^2 - 4k^2$ **12.** *(73)* $36a^2x^2 - k^2$

Factor the trinomials. Always begin by writing the trinomials in descending order of the variables and by factoring out the greatest common factor.

13. (69) $x^2 - x - 20$ **14.** (71) $4x^2 - 4x - 80$ **15.** (71) $2b^2 - 48 - 10b$

16. (71) $-90 - 39x + 3x^2$ **17.** (72) $(a + b)x^2 + 7(a + b)x + 10(a + b)$

18. (71) $pm^2 + 9pm + 20p$ **19.** (71) $5k^2 + 30 + 25k$ **20.** (71) $-x^2 - 8x - 7$

21. (64) Graph on a number line: $-6 \leq x \leq 3$; $D = \{\text{Integers}\}$

22. (61,63) Indicate whether each of the following numbers is a rational number or an irrational number:

(a) $\dfrac{\pi}{2}$ (b) $32.\overline{76}$ (c) $-3\sqrt{3}$ (d) $-\sqrt{121}$

23. (10) Use 10 unit multipliers to convert 25,000 square miles to square kilometers.

24. (67) Use elimination to solve:

$$\begin{cases} 5x - 2y = 3 \\ 2x - 3y = -1 \end{cases}$$

25. (71) Use substitution to solve:

$$\begin{cases} N_P + N_N = 175 \\ N_P + 5N_N = 475 \end{cases}$$

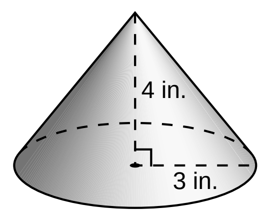
26. (52) Add: $\dfrac{x}{x(x + y)} + \dfrac{1}{x} - \dfrac{y}{x + y}$

Simplify:

27. (66) $3\sqrt{125} + 2\sqrt{45} - \sqrt{50{,}000}$ **28.** (68) $\dfrac{x^{-1} + 1}{yx^{-1} + x}$

29. (31) Solve: $-[2(-3 - k)] = -4(-3) - |-3|k$

30. (72) A right circular cone has a base of radius 3 in. and a height of 4 in., as shown. Find the volume of the right circular cone.



LESSON 74 Scientific Notation

In science courses, it is sometimes necessary to use extremely large numbers and extremely small numbers. For example, to calculate the number of molecules in 1000 liters of gas, it would be necessary to multiply 1000 times 1000 times a very large number such as 26,890,000,000,000,000,000, which represents the number of molecules in a cubic centimeter of gas. Besides requiring a lot of paper, multiplying these numbers in their present form is cumbersome and often leads to errors since it is easy to miscount the number of zeros. If we use a type of mathematical shorthand called **scientific notation,** however, computations such as the above can be performed easily and accurately.

To write a number in scientific notation, the numerator and the denominator are multiplied by the required power of 10 that will place the decimal point immediately to the right of the first nonzero digit in the number (*Note*: We are simply applying the denominator-numerator same-quantity rule). For example, if we wish to write the number

$$0.0000416$$