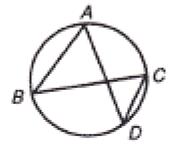
21. Show that: $(1 - \cos^2 x)\csc^2 x + \tan^2 x = \sec^2 x$



CONCEPT REVIEW **22.** If $m\angle ABC$ in the figure shown is 40°, then what is the measure of angle ADC? Why?

23. $(1 + i)^{100}$ equals which of the following quantities?
(*a*) 2^{100} (*b*) -2^{50} (*c*) 2^{50} (*d*) $1 - 2^{100}$

LESSON 34 Identities for the tangent function · Area and volume

34.A Identities for the tangent function

We have used the sum and difference identities for the sine and cosine to develop double-angle and half-angle identities for these functions. Now we will see that we can also use the same identities to develop identities for the tangent function. We repeat the sum and difference identities here. We also repeat the sine-cosine definition of the tangent function as well as the basic Pythagorean identity.

KEY IDENTITIES

(*a*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(*b*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*c*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*d*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(*e*) $\tan A = \dfrac{\sin A}{\cos A}$

(*f*) $\sin^2 A + \cos^2 A = 1$

Example 34.1 Develop an identity for $\tan(A + B)$.

Solution We know that $\tan(A + B)$ equals $\sin(A + B)$ divided by $\cos(A + B)$.

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

There are many forms of tangent identities. We will concentrate on forms in which the first entry in the denominator is the number 1. To change $\cos A \cos B$ to 1, we must divide it by itself. If we do this, we must also divide every other term in the whole expression by $\cos A \cos B$ so that the value of the expression will be unchanged.

$$\tan(A + B) = \frac{\dfrac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \dfrac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B}}{\dfrac{\cancel{\cos A \cos B}}{\cancel{\cos A \cos B}} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$