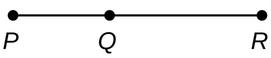
Simplify:

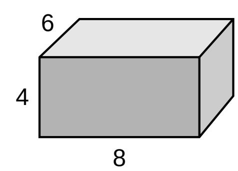
27. (1) $-2\frac{2}{3} + 2\frac{3}{5}$

28. (4) $\dfrac{4\frac{2}{3}}{-3\frac{1}{9}}$

29. (1) The length of $\overline{PQ}$ is $5\frac{1}{3}$ yards. The length of $\overline{QR}$ is $8\frac{5}{12}$ yards. Find PR.



30. (15) Find the surface area of this right rectangular prism. Dimensions are in centimeters.



LESSON 16 More Complicated Evaluations

The procedures discussed in Lesson 14 are also used to evaluate more complicated expressions. The use of parentheses, brackets, and braces is often helpful in preventing mistakes. We will use all of these symbols of inclusion in the following examples.

example 16.1 Evaluate: $-a[-a(p - a)]$ if $p = -2$ and $a = -4$

solution We use parentheses, brackets, and braces as required.

$$-(\quad)\{-(\quad)[(\quad) - (\quad)]\}$$

Now we will insert the numbers inside the parentheses.

$$-(-4)\{-(-4)[(-2) - (-4)]\}$$

Lastly, we simplify.

$$4\{4[2]\} = 4\{8\} = \mathbf{32}$$

example 16.2 Evaluate: $ax[-a(a - x)]$ if $a = -2$ and $x = -6$

solution This time we will not use parentheses. We will replace a with –2, $-a$ with 2, x with –6, and $-x$ with 6.

$$12[2(-2 + 6)]$$

Now we simplify, remembering to begin with the innermost symbol of inclusion.

$$12[2(-2 + 6)] = 12[2(4)] = 12[8] = \mathbf{96}$$

example 16.3 Evaluate: $-b[-b(b - c) - (c - b)]$ if $b = -4$ and $c = -6$

solution We replace b with –4, $-b$ with 4, c with –6, and $-c$ with 6.

$$4[4(-4 + 6) - (-6 + 4)]$$

Now we simplify, remembering to begin within the innermost symbols of inclusion and to multiply before adding.

$$4[4(2) - (-2)] = 4[8 + 2] = 4[10] = \mathbf{40}$$

practice

Evaluate:

a. $-a[-a(p - a)]$ if $p = -4$ and $a = 2$

b. $pa[-p(-a)]$ if $p = -2$ and $a = -4$

c. $-x[-x(x - a) - (a - x)]$ if $x = -2$ and $a = -5$

problem set 16

1. (13) Is the product of 4 positive numbers and 7 negative numbers a positive number or a negative number?

2. (4) (a) What do we call the answer to an addition problem?

(b) What do we call the answer to a subtraction problem?

(c) What do we call the answer to a multiplication problem?

(d) What do we call the answer to a division problem?

3. (2) What do you call polygons in which all sides have the same length and all angles have the same measure?

4. (4) Use one unit multiplier to convert 100 centimeters to inches.

5. (10) Use two unit multipliers to convert 152 square centimeters to square inches.

6. (3) The length of a rectangle is 31 inches. The width of the rectangle is 11 inches. Find the perimeter of the rectangle.

7. (8) The length of a rectangle is 17 feet. The width of the rectangle is 13 feet. Find the area of the rectangle.

8. (8) The radius of a circle is 9 yards. Find the area of the circle.

Evaluate:

9. (14) $x - xy$ if $x = -2$ and $y = -3$

10. (14) $x(x - y)$ if $x = -2$ and $y = -3$

11. (14) $(x - y)(y - x)$ if $x = 2$ and $y = -3$

12. (14) $(x - y) - (x - y)$ if $x = -2$ and $y = 3$

13. (14) $-xa(x - a)$ if $a = -2$ and $x = 4$

14. (14) $(-x + a) - (x - a)$ if $x = -4$ and $a = 5$

15. (14) $(p - x)(a - px)$ if $a = -3$, $p = 2$, and $x = -4$

16. (16) $-a[-a(x - a)]$ if $a = -2$ and $x = 3$

17. (16) $-a[(-x - a) - (x - y)]$ if $a = -3$, $x = 4$, and $y = -5$

Simplify:

18. (12) $-3(-1 - 2)(4 - 5) + 6$

19. (12) $-2 + (-3) - |-5 + 2|3$

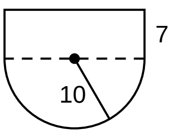
20. (13) $4[2(3 - 2) - (6 - 4)]$

21. (13) $-2(-4) - \{-[-(-6)]\}$

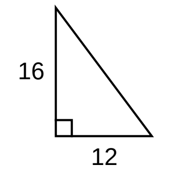
22. (12) $\dfrac{3(-2) - 5}{-3(-2)}$

23. (12) $\dfrac{-3(-6 - 2) + 5}{-3(-2 + 1)}$

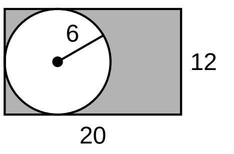
24. (3) Find the perimeter of this figure. Corners that look square are square. Dimensions are in miles.



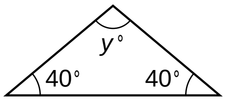
25. *(8)* Find the area of this right triangle. Dimensions are in centimeters.



26. *(8)* Find the area of the shaded portion of this rectangle. Dimensions are in meters.



27. *(2)* Find y.

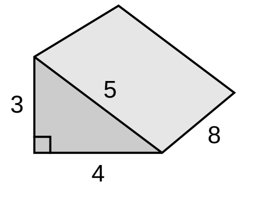


Simplify:

28. *(1)* $1\frac{7}{12} + 5\frac{5}{6} - 4\frac{2}{3}$

29. *(4)* $2\frac{2}{5} \times 11\frac{2}{3}$

30. *(15)* Find the surface area of this right triangular prism. Dimensions are in kilometers.



LESSON 17 *Factors and Coefficients • Terms • The Distributive Property*

17.A factors and coefficients

If the form in which variables and constants are written in an expression indicates that the variables and constants are to be multiplied, we say that the expression is an **indicated product.** If we write

$$4xy$$

we indicate that 4 is to be multiplied by the product of x and y. Each of the symbols is said to be a factor of the expression. Any one factor of an expression or any product of factors of an expression can also be called the **coefficient** of the rest of the expression. Thus, in the expression $4xy$ we can say that

(a) 4 is the coefficient of xy $\quad 4(xy)$

(b) x is the coefficient of $4y$ $\quad x(4y)$

(c) y is the coefficient of $4x$ $\quad y(4x)$

(d) xy is the coefficient of 4 $\quad xy(4)$

(e) $4y$ is the coefficient of x $\quad 4y(x)$

(f) $4x$ is the coefficient of y $\quad 4x(y)$

As mentioned earlier, the value of a product is not affected by the order in which the multiplication is performed, so we may arrange the factors in any order without affecting the value of the expression. Note that we change the order at will in (a) through (f) above.

If the coefficient is a number as in (a) above, we call it a **numerical coefficient,** and if the coefficient consists entirely of variables or letters as in (b), (c), and (d) above, we call it a **literal coefficient.** We need to speak of numerical coefficients so often that we usually drop the adjective *numerical* and use the single word *coefficient*. Thus, in the following expressions

$$4xy \qquad -15pq \qquad 81xmz$$

4 is the coefficient of *xy*, –15 is the coefficient of *pq*, and 81 is the coefficient of *xmz*.

17.B terms

A **term** is an algebraic expression that

1. Consists of a single variable or constant.
2. Is the indicated product or quotient of variables and/or constants.
3. Is the indicated product or quotient of expressions that contain variables and/or constants.

$$4 \qquad x \qquad 4x \qquad \frac{4xy(a + b)}{p} \qquad \frac{3x + 2y}{m}$$

All the expressions above can be called **terms.** The first two are terms because they consist of a single symbol. The third is a term because it is an indicated product of symbols. The fourth and fifth are terms because they are considered to be indicated quotients even though the numerator of the fourth term is an indicated product and the numerator of the fifth term is an indicated sum. **A term is thought of as a single entity that represents or has the value of one particular number.** The word *term* is very useful in allowing us to identify or talk about the parts of a larger expression. For instance, the expression

$$x + 4xym - \frac{6p}{y + 2} - 8$$

is an expression that has four terms. We can speak of a particular term of this expression, say the third term, without having to write out the term in question. The terms of an expression are numbered from left to right, beginning with the number 1. Thus, for the expression above

The first term is $+x$.	The third term is $-\frac{6p}{y + 2}$.
The second term is $+4xym$.	The fourth term is -8.

If we consider that the sign preceding a term indicates addition or subtraction, then the sign is not a part of the term. In this book, we prefer to use the thought of algebraic addition, and thus, most of the time we will consider the sign preceding a term to be a part of the term. But we must be careful.

Let us look at the third term in the expression we are considering.

$$-\frac{6p}{y + 2}$$

If *p* and *y* are given values such that $\frac{6p}{y + 2}$ is a negative number, then $-\frac{6p}{y + 2}$ will be positive. For example, if *p* is equal to –4 and *y* is equal to 1, then the expression has a value of +8.

$$-\frac{6p}{y + 2} = -\frac{6(-4)}{1 + 2} = -(-8) = +8$$

17.C the distributive property

We have noted that the order of adding two real numbers does not change the answer. Also, the order of multiplying two real numbers does not change the answer. We call these two properties or peculiarities of real numbers the **commutative property for addition** and the **commutative property for multiplication.**

Now we will discuss another property of real numbers that is of considerable importance, the **distributive property of real numbers.** If we write

$$4(5 - 3)$$

we indicate that we are to multiply 4 by the algebraic sum of the numbers 5 and –3. A property (or peculiarity) of real numbers permits the value of this product to be found two different ways.

$4(5 - 3)$	$4(5 - 3)$
$4(2)$	$4 \cdot 5 + 4(-3)$
8	$20 - 12$
	8

On the left, we first added 5 and –3 to get 2, and then multiplied by 4 to get 8. On the right, we first multiplied 4 by both 5 and –3, and then added the products 20 and –12 to get 8. Both methods of simplifying the expression led to the same result. **We call this property or peculiarity of real numbers the *distributive property* because we get the same result if we distribute the multiplication over the algebraic addition.**†

DISTRIBUTIVE PROPERTY

For any real numbers a, b, c,

$$a(b + c) = ab + ac$$

It is possible to extend the distributive property so that the extension is applicable to the indicated product of a number or a variable and the algebraic sum of any number of real numbers or variables.

EXTENSION OF THE DISTRIBUTIVE PROPERTY

For any real numbers a, b, c, d, $\ldots$,

$$a(b + c + d + \cdots) = ab + ac + ad + \cdots$$

example 17.1 Use the distributive property to find the value of $4(6 - 2 + 5 - 7)$.

solution We begin by multiplying 4 by each of the terms within the parentheses, and then we add the resultant products.

$$4(6 - 2 + 5 - 7) = 4(6) + 4(-2) + 4(5) + 4(-7)$$
$$= 24 - 8 + 20 - 28 = \mathbf{8}$$

†Note that while multiplication can be distributed over addition, the reverse is not true, for addition cannot be distributed over multiplication. For example,

$$2 + (3 \cdot 5) \neq (2 + 3) \cdot (2 + 5)$$

because $17 \neq 35$