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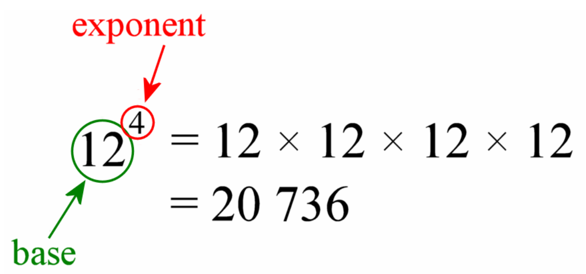
Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{\;3 \times 3 = 9\;}$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

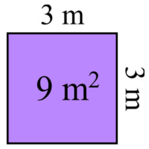
c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square metres and other units of area.



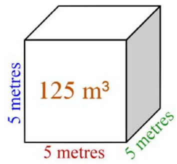
If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square metres.

Notice that the symbol for square metres is **m^2**. This means "**metre × metre**." We are, in effect, squaring the unit *metre* (multiplying the unit of length *metre* by itself)!

The expression $(9\text{ cm})^2$ means 9 cm × 9 cm. We multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 cm^2**. The square centimetre (cm^2) comes from multiplying "**centimetre × centimetre**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometres or square millimetres.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *metre* by itself three times (metre × metre × metre) to get the unit of volume "cubic metre" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometres: A = 12 km × 12 km = ________	**b.** A square with sides 6 m long: A = ________
c. A square with a side length of 6 centimetres: A = ________	**d.** A square with a side with a length of 12 cm: A = ________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with an edge of 2 cm: V = 2 cm × 2 cm × 2 cm = ________	**b.** A cube with edges 10 cm long each: V = ________
c. A cube with edges 1 m in length: V = ________	**d.** A cube with edges that are all 5 m long: V = ________

5. **a.** The perimeter of a square is 40 centimetres. What is its area?

b. The volume of a cube is 64 cubic centimetres. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is the length of one edge?

The powers of 10 are very special —and very easy!

Notice that the exponent tells us *how many zeros* there are in the answer.

$10^1 = 10$	$10^4 = 10\,000$
$10^2 = 10 \times 10 = 100$	$10^5 = 100\,000$
$10^3 = 10 \times 10 \times 10 = 1\,000$	$10^6 = 1\,000\,000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2\,187$, how can you use that result to find 3^8?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35\,184\,372\,088\,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____________ of a _______________ with sides ______ units long.

b. 101^3 gives us the _____________ of a _______________ with edges ______ units long.

c. 2×6^2 gives us the ____________ of two _______________ with sides ______ units long.

d. 4×10^3 gives us the ____________ of ____ _______________ with edges ______ units long.

Puzzle Corner

Make a pattern, called a **sequence**, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, n. What value do you calculate for n^0? Do you think it will come out this way for every base number?

Why or why not?

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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum $b + c$ so that you multiply the numbers b and c separately by a, and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$.
We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot \_\_\_ + 3 \cdot \_\_\_ =$	**b.** $7(50 + 6) = 7 \cdot \_\_\_ + 7 \cdot \_\_\_ =$
c. $4(a + b) = 4 \cdot \_\_\_ + 4 \cdot \_\_\_ =$	**d.** $2(x + 6) = 2 \cdot \_\_\_ + 2 \cdot \_\_\_ =$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the brackets:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Example 4. Now one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

3. Multiply using the distributive property.

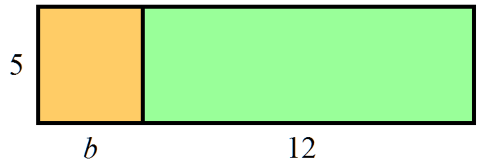
a. $2(3x + 5) =$	**b.** $7(7a + 6) =$
c. $5(4a + 8b) =$	**d.** $2(4x + 3y) =$
e. $3(9 + 10z) =$	**f.** $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	**h.** $8(5 + 2a + 3b) =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times $(b + 12)$.

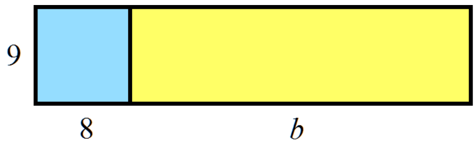
But if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.

Of course, these two expressions have to be equal:



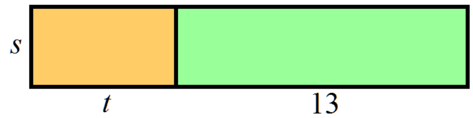
$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

4. Write an expression for the area in two ways, thinking of one rectangle or two.



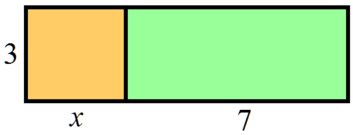
a. 9(_____ + _____) and

9 · _____ + 9 · _____ =

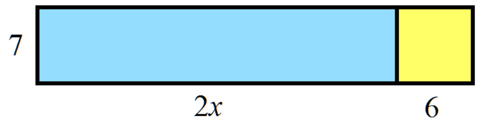


b. s(_____ + _____) and

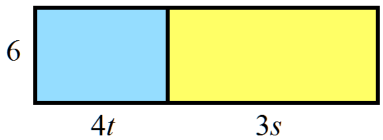
s · _____ + s · _____ =



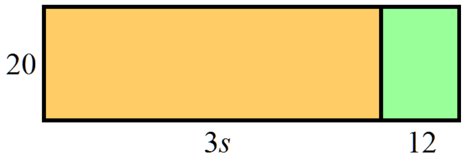
c. ____ (_____ + _____) and



d.



e.



f.

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Using Two Variables

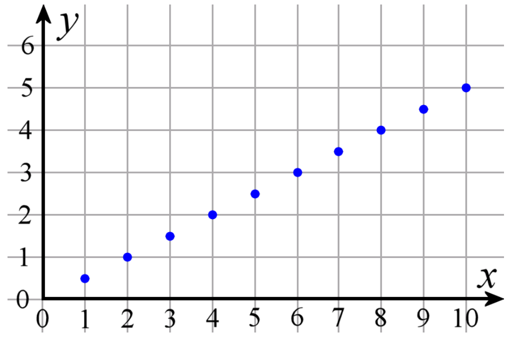
Often in mathematics—and in real life—we study the relationship between two variables.

Example 1. The equation $y = \frac{1}{2}x$ has two variables, *y* and *x*.

There are many values of *x* and *y* that make that equation true. For example, when *x* is 4, then *y* is (1/2) · 4 = 2. Some of the values of *x* and *y* are listed below.

x	1	2	3	4	5
y	½	1	1 ½	2	2 ½

x	6	7	8	9	10
y	3	3 ½	4	4 ½	5



We can plot or graph these (*x*, *y*) pairs as points in the coordinate grid.
These ordered pairs actually are a **function**. We will not study the exact definition of a function here, but you can think of a function as a relationship between two variables.

In this lesson, you will study only **linear functions**. The word "linear" comes from the fact that the graphs of those functions look like a *line*. There exist many other, different kinds of functions as well.

Example 2. One towel costs $4. If you buy 17 towels, the cost is 17 · $4 = $68.
In this situation, we are interested in two variables whose values can change:

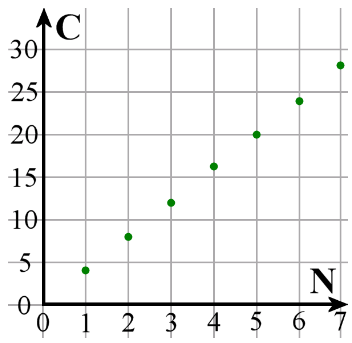
1. **The number of towels** a person buys is a variable. (It can vary!) Let's denote the number of towels by N.
2. **The total** cost varies according to how many towels are bought. Let C be the cost.

There is a very simple relationship between N and C: **C = N · $4**
(This means the total cost *is* the number of towels times $4.)

This is normally written as **C = 4N** because in algebra we write the number in front of the variable (not vice versa), and we omit the multiplication sign between a number and a variable.

The table below shows some *possible* values of C and N.

(x)	N	1	2	3	4	5	6	7	10	15	20
(y)	C	4	8	12	16	20	24	28	40	60	80



From this table, we get lots of number pairs. Some of them are plotted on the coordinate grid you see on the right.

You may have seen coordinate grids that have *x* and *y*-axis. This time we will label our axes N and C, according to the names of the variables. If this seems confusing, think of the variable N as the "*x*", and the variable C as the "*y*".

In this situation, we think of the variable N as the *independent variable*, and the variable C as the *dependent variable*, because its value *depends* on the value of N according to the given equation (C = 4N). In other words, we let the value of N vary (sort of independently), and the values of C are what we calculate or "observe," noticing how they depend on the value of N.

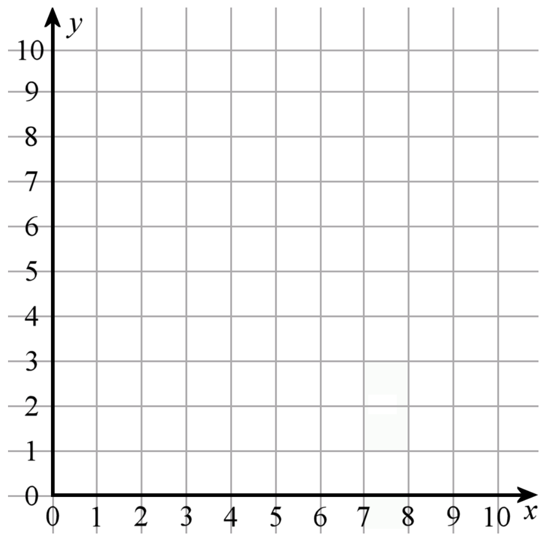
The independent variable is *always* plotted on the horizontal axis.

We *could* look at this situation just the opposite way also: let the cost be the independent variable, and study how the number of towels depends on that. Then, we would plot C on the horizontal axis, and calculate N using an equation that depends on C (it would be N = C/4).

1. Calculate the values of y according to the equation $y = x + 2$.

x	0	1	2	3	4	5	6	7	8
y	2	3	4						

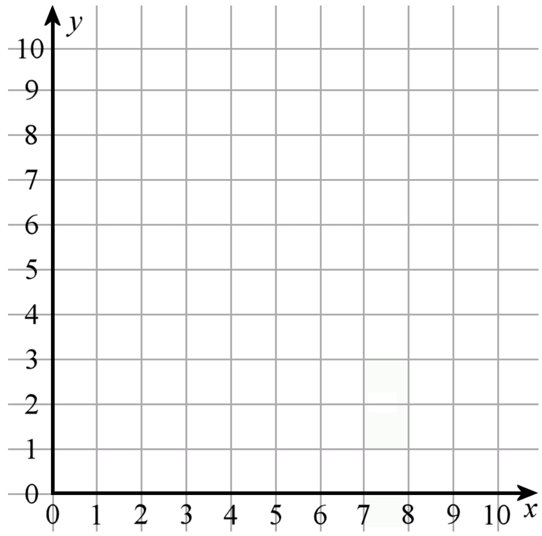
Now, plot the points.



2. Calculate the values of y according to the equation $y = 8 - x$.

x	0	1	2	3	4	5	6	7	8
y	8								

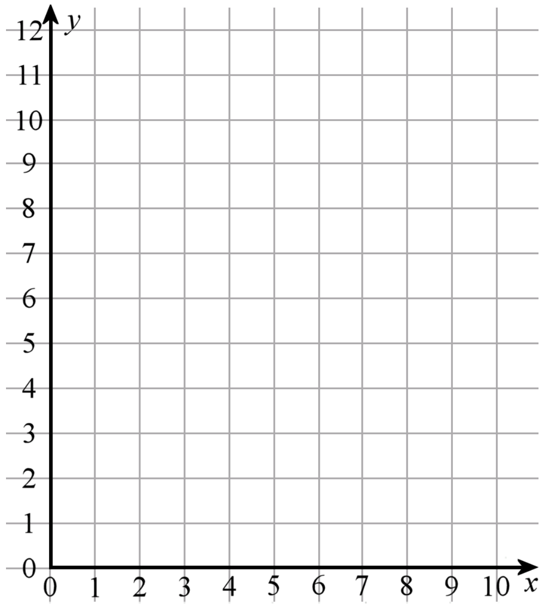
Now, plot the points.



3. Calculate the values of y according to the equation $y = 2x - 1$.

x	1	2	3	4	5	6
y						

Now, plot the points.



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Problem Solving with Decimals

Example 1. Martha jogs 0.8 kilometre every day. How many days will it take for her to jog a distance of 20 kilometres?

We could divide 20 km by 0.8 km to find out how many times 0.8 "fits" into 20. However, there is also another way: we can solve it with *mental maths*.

Notice, 0.8 goes evenly into 4, and 4 goes evenly into 20.

0.8 fits five times into 4 (because $5 \cdot 0.8 = 4$). And, 4 fits five times into 20.

So, 0.8 fits into 20 exactly $5 \cdot 5 = 25$ times.

Martha will have jogged 20 kilometres in 25 days.

Example 2. If you divide a paper that is 21.25 cm wide into three equally-wide columns. How wide are the columns?

We divide 21.25 cm by three. Since the width of 21.25 cm is given to *two* decimal places, it is reasonable to also give the answer to *two* decimal places, so divide until there are *three* decimals in the quotient, and then round to the nearest hundredth.

```
      0 7.0 8 3
   ------------
 3 ) 2 1.2 5 0
    -2 4
       1 0
      -  9
         1
```

$21.25 \div 3 \approx 7.08$, so the columns are about 7.08 cm wide.

If you have to actually measure these columns using a standard ruler, then it would be reasonable to give the answer as 7.1 cm.

In all of the problems, give your answer to a meaningful accuracy, especially when the division is not even.

1. Jack, John and Jerry shared a prize of $200 equally.
   How much did each one get?

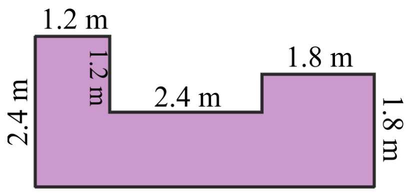
2. A student textbook weighs 0.4 kg. How many of those can you pack
   into a suitcase so that the total weight is 18 kg?

3. These are the quiz results of the Spanish class:
   21 15 18 29 19 34 39 21 11 8 15 28 15 11 12.
   Find the average.

To calculate the average of a set of numbers:

1. Add all of the numbers.
2. Divide the sum by the number of the data entries.

4. Find the area and perimeter of this shape.



5. Kitchen Delight makes blenders. Each blender weighs 1.2 kg. The shipping company allows no more than 40 kg of weight in each shipping crate. How many blenders can be packed into each shipping crate?

6. Find the unit prices for the following items. Round to the nearest cent. Use the space below for calculations.

Item and price	Unit price	What would this cost…?	
5 L of orange juice for $15.65		2.3 L of orange juice	
3 kg of chicken for $13.25		5.7 kg of chicken	
4 kg of bananas for $7.90		2.5 kg of bananas	

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Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **metre**. For weight, it is the **gram**. And for volume, it is the **litre**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centilitre is 1/100 part of a litre (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1 000
hecto-	h	100
deka-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometre	km	1 000 metres
hectometre	hm	100 metres
decametre	dam	10 metres
metre	m	(the basic unit)
decimetre	dm	1/10 metre
centimetre	cm	1/100 metre
millimetre	mm	1/1000 metre

Unit	Abbr	Meaning
kilogram	kg	1 000 grams
hectogram	hg	100 grams
dekagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kilolitre	kl	1 000 litres
hectolitre	hl	100 litres
dekalitre	dal	10 litres
litre	L	(the basic unit)
decilitre	dl	1/10 litre
centilitre	cl	1/100 litre
millilitre	ml	1/1000 litre

1. Write these amounts using the basic units (metres, grams, or litres) by "translating" the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0.03 m (since "centi" means "hundredth part").

a. 3 cm = *3/100 m* = *0.03 m*
5 mm = ________ m = ________ m
7 dl = ________ L = ________ L

b. 2 cg = ________ g = ________ g
6 ml = ________ L = ________ L
1 dg = ________ g = ________ g

2. Write the amounts in basic units (metres, grams, or litres) by "translating" the prefixes.

a. 3 kl = ________ L
8 dag = ________ g
6 hm = ________ m

b. 2 dam = ________ m
9 hl = ________ L
7 kg = ________ g

c. 70 km = ________ m
5 hg = ________ g
8 dal = ________ L

3. Write the amounts with derived units (units with prefixes) and a single-digit number.

a. 3 000 g = 3 kg
800 L = 8 ______
60 m = 6 ______

b. 0.01 m = ________ ______
0.2 L = ________ ______
0.005 g = ________ ______

c. 0.04 L = ________ ______
0.8 m = ________ ______
0.007 L = ________ ______

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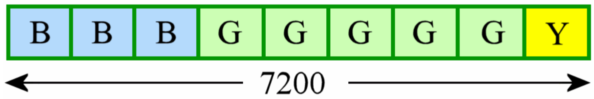
Ratio Problems and Bar Models 1

Often, ratio problems become easy by drawing a **bar model**.

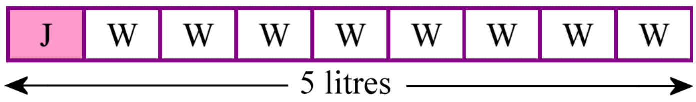
The ratio of blue shirts to green shirts to yellow shirts is 3 to 5 to 1. If there are 7 200 shirts in all, how many of them are of each colour?



Look at the bar model. There are a total of 7 200 shirts.
We draw 3 “blocks” for the blue shirts, 5 “blocks” for the green shirts and 1 “block” for the yellow shirts to show the ratio of **3 : 5 : 1**. It is obvious one “block” means 7 200 ÷ 9 = 800 shirts. So there are a total of 2 400 blue shirts, 4 000 green shirts and 800 yellow shirts.

Juice concentrate is mixed with water in a ratio of 1:8. If you want to make 5 litres of juice, how much concentrate and how much water do you need?

Let’s draw a bar model. (In reality, of course, the juice and water mix, but for the purpose of calculating, this model is helpful.)



There are a total of 9 equal parts, so we simply divide 5 litres by 9. First, change 5 litres to 5 000 millilitres, and then divide: 5 000 ml ÷ 9 ≈ 555.56 ml.

However, that is way too accurate. Measuring cups do not normally let us measure to the nearest millilitre, and not even to the nearest 10 millilitres, so let’s round this to the nearest 50 ml to get 550 ml.

So we need 550 ml of juice concentrate and 5 000 ml − 550 ml = 4 450 ml of water.

1. A factory makes shirts in a ratio of 1:3:3:1 for the sizes S, M, L and XL, respectively.

   a. Draw a bar model. What is the ratio of small (S) shirts to the total number of shirts?

   b. In a batch of 1 000 shirts, how many of them are of each size?

2. The instructions on a box of juice concentrate say to mix 2 parts of concentrate to 5 parts of water.

   a. If you want to make 3 litres of juice, how much concentrate and how much water do you need?

   b. Let’s say that you have 1/2 litre of concentrate left. According to the instructions, how much water would you need to add to that?

   How much diluted juice does this make?

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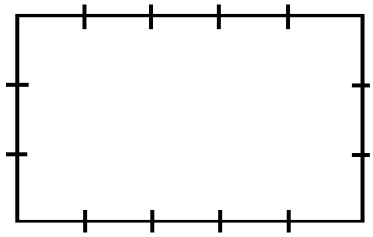
Aspect Ratio

You might have heard about the aspect ratio of the screens of televisions, computer monitors and other monitors. The aspect ratio is simply **the ratio** of a **rectangle's width to its height or length.**

If the rectangle is "standing up," it is often easier to think and talk about width and height. If it is laid on the ground, then we usually talk about its width and length.

Example. A rectangle's width and height are in a ratio of 5:3. This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?



Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since 64 ÷ 16 = 4, each part is 4 cm long.

Therefore, the rectangle's width is 5 · 4 cm = 20 cm, and its height is 3 · 4 cm = 12 cm.

1. The width and height of a rectangle are in a ratio of 9:2.

   a. Draw the rectangle, and divide its width and length into parts according to its aspect ratio.

   b. If the rectangle's perimeter is 220 cm, find its width and its height.

2. A rectangle's width is three times its length, and its perimeter is 120 mm. Find the rectangle's width and its length.

3. Find the aspect ratio of each rectangle:

   a. a rectangle whose height is 2/5 of its width

   b. a rectangle whose height is five times its width

   c. a square

4. The door of a refrigerator is 4/9 as wide as it is tall.

   a. What is the ratio of the door's width to its height?

   b. If the door is 54 cm wide, how tall is it?

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Percentage of a Number (Mental Maths)

100% of something means *all* of it. 1% of something means 1/100 of it.
Since one percent means "a hundredth part," calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions**!
How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg. **To find 1% of something (1/100 of something), divide by 100.** Do you remember how to divide by 100 in your head? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.
To find 2% of some quantity, first find 1% of it, and double that. For example, let's find 2% of \$6. Since 1% of \$6 is \$0.06, then 2% of \$6 is \$0.12.
To find 10% of some quantity, divide by 10. Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity! For example, 10% of \$780 is \$78. And 10% of \$6.50 is \$0.65. (To divide by 10 in your head, just move the decimal point one place to the left.)
Can you think of a way to find 20% of a number? ____

1. Find 10% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

2. Find 1% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

3. One percent of Mother's pay cheque is \$22. How much is her total pay cheque?

4. Fill in the table. Use mental maths.

percentage ↓ number →	**1 200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental maths with percentages:

Mental Maths and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _______.	50% of 244 is _________.
10% is $\frac{1}{\square}$. To find 10% of a number, divide by ________.	10% of 47 is __________.
1% is $\frac{1}{\square}$. To find 1% of a number, divide by ________.	1% of 530 is __________ .
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • **First find __________% of the number, and** • **then multiply by 2, 3, 4, 6, 7, 8, or 9.**	10% of 120 is __________. 30% of 120 is __________. 60% of 120 is __________.

6. Find the percentages. Use mental maths.

a. 10% of 60 kg __________ 20% of 60 kg __________	**b.** 10% of $14 __________ 30% of $14 __________	**c.** 10% of 5 m __________ 40% of 5 m __________
d. 1% of $60 __________ 4% of $60 __________	**e.** 10% of 110 cm __________ 70% of 110 cm __________	**f.** 1% of $1 330 __________ 3% of $1 330 __________

7. David pays a 20% income tax on his $2 100 salary.

a. How many dollars is the tax?

b. How much money does he have left after paying the tax?

c. What percentage of his salary does he have left?

8. Nancy pays 30% of her $3 100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of $55. Peter's solution: 10% of $55 is $5.50 So, I subtract 100% − $5.50 = $94.50	**b.** Find 6% of $1 400. Patricia's solution: 1% of $1 400 is $1.40. So, 6% is six times that, or $8.40.

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Finding the Total When the Percentage Is Known

Use a bar model to find the unknown total when you know the percentage and the quantity.

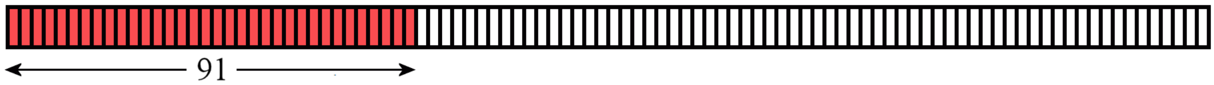
Example 1. If 32 red marbles make up 4/5 of the total number of marbles, how many marbles are there in all?

Look at the bar model. We have drawn the marbles as divided into 5 equal "blocks." Four of those five blocks make up a total of 32 marbles. So, one block, or 1/5 of the marbles, is 8 marbles. From that it is easy to calculate the total: $5 \cdot 8 = 40$ marbles.

The same reasoning works if the part of the marbles is given as a *percentage* instead of as a fraction:

Example 2. If 91 red marbles is 35% of the total number of marbles, how many marbles are there in all?

In the model, we need 100 little "blocks" with 35 of them coloured (since 35/100 of the marbles are red.)



The calculation is done the same way: If 35 "blocks" or 35% make up 91 marbles, then one "block", or one percent, is $91 \div 35 = 2.6$. Then, to find the total, simply multiply that number by 100: $2.6 \cdot 100 = 260$.

1. Margie gave away 40 marbles, which was 20% of the marbles that she had.
   How many marbles did Margie have at first?
   Hint: Instead of 100 blocks, you can use 5 blocks, each representing 20% or 1/5.

2. Emma cut down the amount of sugar in a recipe by 75%.
   Now, she uses only 1/2 cup of sugar.
   How much sugar did the recipe call for originally?
   Hint: Instead of 100 blocks, you can use 4 blocks, each representing 25%.

3. When Eric bought a guitar for $90, he used up 12% of the money he had.
   How much money did he have at first?

Example 3. A phone was discounted by 40% and now costs $72. What was the price before the discount?

The cost now, $72, represents **60%** of the original total—not 40%.

We can find 10% of the original price by dividing $72 ÷ 6 = $12. And from that, 100% of the price is 10 times that, or $120. If this confuses you, draw a bar model with 10 parts, each representing 10% of the original price.

4. A dress was discounted by 20%.
The discounted price is $24.
What was the price before the discount?

5. A concert ticket was discounted by 60%.
The discounted price is $21.60.
What was the original price?

6. Joe spent 72% of his money, and now he has $56 left.
How much did Joe have to begin with?

7. Crystal spent 52% of her money and now she has $120 left.
How much did she spend?

8. Uncle Jack raises two different breeds of cows on his farm. Of his cows, 28% are Black Angus and the rest are Hereford. If he has 420 Black Angus cows, how many Herefords does he have?

9. A survey found out that 16% of the people who had bought a certain brand of coffee grinder were unhappy with it. If there were 126 people who *were* happy with it, then how many people in total had bought that brand?

Puzzle Corner

One calculator is discounted by 30% and now costs $42.
Another is discounted by 25% and now it also costs $42.
Which calculator had the cheaper original price? How much cheaper?