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Foreword

Math Mammoth Grade 5, International Version, comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 5, only customised for international audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the fifth grade standards in your country.

The international version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses only metric measurement units.
- The spelling conforms to British international standards.
- The pages are formatted for A4-size paper.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction, and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorising two-dimensional figures (especially triangles);

The year starts out with a study of the basic operations, some algebraic concepts, and primes and divisibility. In chapter 2, we go on to study place value, large numbers and the usage of the calculator.

In chapter 3, students solve simple equations with the help of a pan balance. Then they learn to solve a variety of word problems using the bar model as a visual aid.

Chapter 4 is all about decimals and decimal arithmetic. Several lessons here focus on mental maths strategies based on place value.

The last chapter in this part A is on graphing. Students encounter the coordinate plane and simple number patterns that are plotted as points on the grid. They also plot and read line graphs.

In part 5-B, students study more decimal arithmetic, all fraction operations, and geometry.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at <https://www.mathmammoth.com/userguides/>.

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at <https://www.mathmammoth.com/videos/>. There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don’t automatically assign all the exercises. Use your judgement, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at <https://www.mathmammoth.com/practice/>. This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at <https://www.mathmammoth.com/videos/>

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

| Grade level | School days | Days for tests and revisions | Lesson pages | Days for the student book | Pages to study per day | Pages to study per week |
|---------------|-------------|------------------------------|--------------|---------------------------|------------------------|-------------------------|
| 5-A | 88 | 10 | 178 | 78 | 2.28 | 11.4 |
| 5-B | 92 | 10 | 188 | 82 | 2.29 | 11.5 |
| Grade 5 total | 180 | 20 | 366 | 160 | 2.29 | 11.4 |

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of “pages to study per day”, **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

| Grade level | Number of school days | Days for tests and revisions | Lesson pages | Days for the student book | Pages to study per day | Pages to study per week |
|---------------|-----------------------|------------------------------|--------------|---------------------------|------------------------|-------------------------|
| 5-A | | | 178 | | | |
| 5-B | | | 188 | | | |
| Grade 5 total | | | 366 | | | |

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

| Worktext 5-A | |
|--------------|----------------|
| Chapter 1 | 21 days |
| Chapter 2 | 12 days |
| Chapter 3 | 9 days |
| Chapter 4 | 18 days |
| Chapter 5 | 11 days |
| TOTAL | 71 days |

| Worktext 5-B | |
|--------------|----------------|
| Chapter 6 | 20 days |
| Chapter 7 | 15 days |
| Chapter 8 | 20 days |
| Chapter 9 | 12 days |
| TOTAL | 67 days |

In general, 1st-2nd graders might spend 25-40 minutes a day on maths. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

Working space, the usage of additional paper and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students use extra paper when necessary. They can use it, not only for the “long” algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. $200 + \underline{\quad} = 1000$). Typically, I have intended that such exercises should be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, “Cumulative Revision, Chapter 4” includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at <https://www.mathmammoth.com/practice/> has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think “outside of the box” or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)

<https://www.homeschoolmath.net/teaching/math-stars.php>

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

<https://l.mathmammoth.com/challengingproblems>

Another idea: you can find puzzles online by searching for “brain puzzles for kids,” “logic puzzles for kids” or “brain teasers for kids.”

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at <https://www.mathmammoth.com/faq-lightblue>

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: The Four Operations

Introduction

We start fifth grade by studying the four basic operations. The topics include the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought in the beginning portion of the chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students determine which expression matches the given word problem, and write simple expressions for word problems, using the correct order of operations. Thus, they are learning how to represent a situation symbolically, which is a very important step in using mathematics to solve problems.

We also briefly study the concept of an equation, and students solve simple equations in several lessons.

Next, we revise multi-digit multiplication, starting with partial products (including a geometric visualisation), and then going on to the standard multiplication algorithm with more digits than in 4th grade.

Then it is time for long division, especially practising long division with two-digit divisors. We also study why long division works, in the optional lesson *Long Division and Repeated Subtraction*. You can use the lesson as time allows and according to student interest. Throughout the lessons there are also word problems to solve.

The lessons for long multiplication often ask the student to estimate the answer before calculating. The lessons for long division ask for the student to check the answer by multiplying. Both of these methods serve the same purpose: to help them gauge whether the calculation is correct. Too often, students simply calculate something and hurry on by, without paying attention to their own work. We need to foster in them a sense of carefulness with calculations, and the habit of checking one's own work for accuracy. If necessary, assign less problems (especially similar calculations) so that students have time to think about and check their answers.

Lastly, we study the topics of divisibility, primes, and factoring. Students revise or learn the common divisibility rules for 2, 3, 4, 5, 6, 9, and 10. In prime factorisation, we use factor trees. The topic of finding factors is revision from 4th grade. Prime factorisation is a new topic; it is also studied in 6th grade.

Although the chapter is named "The Four Operations," the idea is not to practise each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are developing the students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, and "undo" operations.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file). Please add one day to the pacing for the test if you use it.

| The Lessons in Chapter 1 | page | span | suggested pacing | your pacing |
|---|------|---------|------------------|-------------|
| Warm Up: Mental Maths | 13 | 2 pages | 1 day | |
| The Order of Operations | 15 | 2 pages | 1 day | |
| Equations | 17 | 2 pages | 1 day | |
| Revision: Addition and Subtraction | 19 | 3 pages | 1 day | |
| Revision: Multiplication and Division | 22 | 3 pages | 1 day | |
| Partial Products, Part 1 | 25 | 3 pages | 1 day | |
| Partial Products, Part 2 | 28 | 3 pages | 1 day | |
| The Multiplication Algorithm | 31 | 5 pages | 2 days | |
| More Multiplication | 36 | 5 pages | 2 days | |
| Revision of Long Division | 41 | 3 pages | 1 day | |

| The Lessons in Chapter 1 | page | span | suggested pacing | your pacing |
|---|------|------------|------------------|-------------|
| A Two-Digit Divisor | 44 | 3 pages | 1 day | |
| More Long Division | 47 | 4 pages | 1 day | |
| Division with Mental Maths | 51 | 3 pages | 1 day | |
| Long Division and Repeated Subtraction (optional) | 53 | (5 pages) | (2 days) | |
| Divisibility and Factors | 58 | 3 pages | 1 day | |
| More on Divisibility | 61 | 2 pages | 1 day | |
| Primes and Finding Factors | 63 | 3 pages | 1 day | |
| Prime Factorisation | 66 | 5 pages | 2 days | |
| Chapter 1 Revision | 71 | 3 pages | 1 day | |
| Chapter 1 Test (optional) | | | | |
| TOTALS | | 57 pages | 21 days | |
| with optional content | | (62 pages) | (23 days) | |

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

<https://l.mathmammoth.com/gr5ch1>



Warm-up: Mental Maths

| | |
|---|---|
| <p>Add in parts.</p> <p>$57 + 34 = ?$</p> <p>Add the tens: $50 + 30 = 80$. Add the ones: $7 + 4 = 11$. Lastly, add the two sums: $80 + 11 = 91$.</p> | <p>Use rounded numbers, then correct the error.</p> <p>$29 + 18 = ?$</p> <p>29 is close to 30, and 18 is close to 20. $30 + 20 = 50$. But that is 3 too many, so the correct answer is 47.</p> |
| <p>Subtract in parts.</p> <p>$81 - 34 = ?$</p> <p>Subtract 30 first: $81 - 30 = 51$. Then subtract four: $51 - 4 = 47$.</p> | <p>Use rounded numbers, then correct the error.</p> <p>$75 - 39 = ?$</p> <p>39 is close to 40, so subtract $75 - 40 = 35$. You subtracted one too many, so add one to get the correct answer 36.</p> |

1. Add and subtract using the tricks explained above.

| | | |
|--|--|---|
| <p>a.</p> <p>$19 + 19 = \underline{\hspace{2cm}}$</p> <p>$28 + 47 = \underline{\hspace{2cm}}$</p> | <p>b.</p> <p>$19 + 19 + 57 = \underline{\hspace{2cm}}$</p> <p>$44 + 12 + 29 = \underline{\hspace{2cm}}$</p> | <p>c.</p> <p>$100 + 200 + 2000 + 5500 = \underline{\hspace{2cm}}$</p> <p>$400 + 12\ 000 + 5000 + 320 = \underline{\hspace{2cm}}$</p> |
| <p>d.</p> <p>$33 - 17 = \underline{\hspace{2cm}}$</p> <p>$81 - 47 = \underline{\hspace{2cm}}$</p> | <p>e.</p> <p>$34 - 19 + 12 = \underline{\hspace{2cm}}$</p> <p>$85 - 12 + 55 = \underline{\hspace{2cm}}$</p> | <p>f.</p> <p>$1500 - 250 - 250 = \underline{\hspace{2cm}}$</p> <p>$400 - 7 - 40 - 100 = \underline{\hspace{2cm}}$</p> |

2. A track has four legs of different lengths: (a) 1 km 200 m, (b) 700 m, (c) 1 km 500 m, and (d) 900 m. What is the total length of the track?

Hint: "kilo" in kilometre refers to one thousand.

3. A cold front just arrived, and the temperature dropped 17 degrees. It is now 11°C . What was it before?
4. Four crates of apples weigh a total of 56 kg. The first one weighs 12 kg, the second one 15 kg, and the third one 22 kg. Find the weight of the fourth crate of apples.
5. Solve in your head.

| | | |
|--|--|--|
| a. $127 + \underline{\hspace{2cm}} = 200$ | b. $250 + \underline{\hspace{2cm}} + 300 = 760$ | c. $\underline{\hspace{2cm}} - 34 = 56$ |
|--|--|--|

6. Multiply.

| | | |
|---|--|--|
| a. $20 \times 6 = \underline{\hspace{2cm}}$ | b. $10 \times 35 = \underline{\hspace{2cm}}$ | c. $400 \times 500 = \underline{\hspace{2cm}}$ |
| $200 \times 6 = \underline{\hspace{2cm}}$ | $100 \times 35 = \underline{\hspace{2cm}}$ | $60 \times 80 = \underline{\hspace{2cm}}$ |
| $200 \times 600 = \underline{\hspace{2cm}}$ | $20 \times 100 = \underline{\hspace{2cm}}$ | $100 \times 430 = \underline{\hspace{2cm}}$ |

7. Continue the patterns for the next five numbers.

a. 60, 120, 180, 240, ...

b. 1080, 960, 840, 720, ...

c. 130, 170, 210, 250, ...

8. Estimate the cost of buying two skirts for \$26.95 and three pairs of socks for \$3.29 each. (Use rounded numbers.)

| | |
|--|--|
| <p>Multiply part-by-part</p> <p>Multiply ones, tens, and hundreds separately. Add.</p> <p>$3 \times 62 = \underline{3 \times 60} + \underline{3 \times 2} = 186$</p> | <p>5 times a number</p> <p>Find 10 times half of the number.</p> <p>$5 \times 28 = \underline{10 \times 14} = 140.$</p> |
| <p>9 times a number</p> <p>Find 10 times a number and subtract that number once.</p> <p>$9 \times 55 = \underline{10 \times 55 - 55}$ $= 550 - 55 = 495$</p> | <p>11 times a number</p> <p>Find 10 times the number, and then add that number.</p> <p>$11 \times 38 = \underline{10 \times 38 + 38}$ $= 380 + 38 = 418$</p> |

9. Multiply using the “tricks” explained above.

a. $5 \times 26 = \underline{\hspace{2cm}}$

b. $5 \times 43 = \underline{\hspace{2cm}}$

c. $6 \times 41 = \underline{\hspace{2cm}}$

d. $5 \times 107 = \underline{\hspace{2cm}}$

e. $9 \times 15 = \underline{\hspace{2cm}}$

f. $9 \times 32 = \underline{\hspace{2cm}}$

g. $7 \times 205 = \underline{\hspace{2cm}}$

h. $3 \times 211 = \underline{\hspace{2cm}}$

i. $11 \times 25 = \underline{\hspace{2cm}}$

j. $11 \times 18 = \underline{\hspace{2cm}}$

k. $4 \times 32 = \underline{\hspace{2cm}}$

l. $9 \times 109 = \underline{\hspace{2cm}}$

[This page is intentionally left blank.]

The Multiplication Algorithm

An *algorithm* is a step-by-step method for solving a particular kind of problem.

In this lesson we practise **the standard multiplication algorithm**, which you already know from 4th grade.

This algorithm is based on multiplying in parts. For example, 7×648 is done in three parts: 7×600 , 7×40 , and 7×8 .

At each step, you may need to regroup and add.

$$\begin{array}{r} 648 \\ \times 7 \\ \hline \end{array}$$

$7 \times 8 = 56$

$$\begin{array}{r} 648 \\ \times 7 \\ \hline \end{array}$$

$7 \times 40 + 5 = 33$

$$\begin{array}{r} 648 \\ \times 7 \\ \hline \end{array}$$

$7 \times 600 + 3 = 45$

1. Revise your multiplication skills.

a.
$$\begin{array}{r} 415 \\ \times 8 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 877 \\ \times 8 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1752 \\ \times 7 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 2615 \\ \times 4 \\ \hline \end{array}$$

The process is the same with more digits. Study the example.

$$\begin{array}{r} 61359 \\ \times 5 \\ \hline \end{array}$$

$5 \times 9 = 45$

$$\begin{array}{r} 61359 \\ \times 5 \\ \hline \end{array}$$

$5 \times 5 + 4 = 29$

$$\begin{array}{r} 61359 \\ \times 5 \\ \hline \end{array}$$

$5 \times 3 + 2 = 17$

$$\begin{array}{r} 61359 \\ \times 5 \\ \hline \end{array}$$

$5 \times 1 + 1 = 6$

$$\begin{array}{r} 61359 \\ \times 5 \\ \hline \end{array}$$

$5 \times 6 = 30$

2. Multiply 5- and 6-digit numbers.

a.
$$\begin{array}{r} 17552 \\ \times 7 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 27805 \\ \times 3 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 144123 \\ \times 5 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 270814 \\ \times 3 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 51620 \\ \times 9 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 239313 \\ \times 4 \\ \hline \end{array}$$

Estimate before you multiply. Then compare your estimated result with the final result, and that way you may catch some gross errors.

$$3 \times 21\,578 = ?$$

Round 21 578 in such a way that you can easily multiply in your head. It makes sense to round it to 22 000.

Estimate: $3 \times 22\,000 = 66\,000$

The exact result is 64 734. The estimate is quite close.

$$\begin{array}{r} 1\ 2\ 2 \\ 2\ 1\ 5\ 7\ 8 \\ \times \qquad \qquad 3 \\ \hline 6\ 4\ 7\ 3\ 4 \end{array}$$

3. First estimate, by rounding the number in such a way that you can multiply in your head. Then multiply. Check that your final answer is reasonably close to your estimate.

a. **Estimate:** $5 \times 8871 \approx$ _____

Calculate exactly:

$$\begin{array}{r} 8\ 8\ 7\ 1 \\ \times \qquad \qquad 5 \\ \hline \end{array}$$

b. **Estimate:** $4 \times 22\,399 \approx$ _____

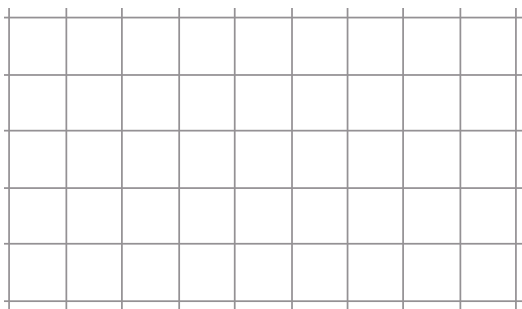
Calculate exactly:

$$\begin{array}{r} 2\ 2\ 3\ 9\ 9 \\ \times \qquad \qquad 4 \\ \hline \end{array}$$

c. **Estimate:** $7 \times 87\,240$

\approx _____

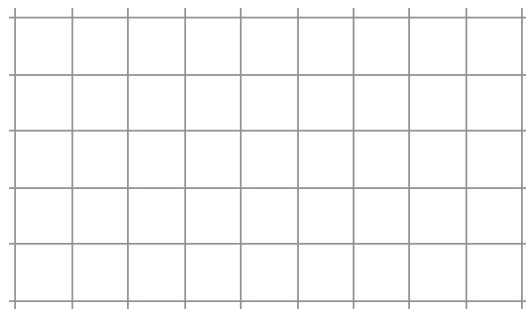
Calculate exactly:



d. **Estimate:** $4 \times 212\,788$

\approx _____

Calculate exactly:



4. Jenny's estimate for the problem $3 \times 173\,039$ is quite far from her final answer. Figure out where Jenny makes an error or errors.

Jenny's estimate:

$$\begin{aligned} &3 \times 173\,039 \\ \approx &3 \times 170\,000 \\ = &510\,000 \end{aligned}$$

Jenny's calculation:

$$\begin{array}{r} 1\ 2 \\ 1\ 7\ 3\ 0\ 3\ 9 \\ \times \qquad \qquad 3 \\ \hline 4\ 2\ 9\ 0\ 1\ 7 \end{array}$$

Multiplying with money amounts is done the same way as with whole numbers: we multiply as if there was no decimal point.

Continue the example on the right.

Lastly, put the decimal point in the answer to mark the two digits for the cents.

$$\begin{array}{r} 214.\overset{4}{1}8 \\ \times 5 \\ \hline 90 \end{array}$$

5. Multiply.

| | | | |
|---|---|--|--|
| <p>a.</p> $\begin{array}{r} \$22.72 \\ \times 8 \\ \hline \end{array}$ | <p>b.</p> $\begin{array}{r} \$81.50 \\ \times 4 \\ \hline \end{array}$ | <p>c.</p> $\begin{array}{r} \$345.25 \\ \times 6 \\ \hline \end{array}$ | <p>d.</p> $\begin{array}{r} \$712.90 \\ \times 5 \\ \hline \end{array}$ |
|---|---|--|--|

6. Emma bought three lamps for \$31.75 each, and paid with \$100. What was her change?

- a. Write a **single expression** for this situation that includes two operations. Remember to consider the order of operations.
- b. Find the answer (her change).

7. First estimate the total cost by rounding the price. Should you round it to the nearest dollar or to the nearest ten dollars? That depends on how well you can multiply in your head. Then find the exact cost.

| | |
|--|---|
| <p>a. Jack bought two train sets for \$56.55 each.</p> <p>Estimate: _____</p> <div style="border: 1px solid gray; width: 100%; height: 100%; display: flex; flex-wrap: wrap;"> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> </div> | <p>b. The Internet is \$128.95 per month. What does it cost for 6 months?</p> <p>Estimate: _____</p> <div style="border: 1px solid gray; width: 100%; height: 100%; display: flex; flex-wrap: wrap;"> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> <div style="width: 50%; height: 50%;"></div> </div> |
|--|---|

Revise how to multiply a two-digit number by a two-digit number.

Estimate: Round $29 \approx 30$
and $75 \approx 70$. Then, 29×75
 $\approx 30 \times 70 = 2100$.

Notice! We rounded 75 *down* to 70.

Why? Because if we rounded both factors up, we would overestimate the answer. When multiplying, it often works out better to round one factor down and the other up.

Let's check that out: If we rounded 29 and 75 to 30 and 80, our estimation would be $30 \times 80 = 2400$. That is not nearly as good an estimate as 2100. (The exact answer is 2175.)

$$\begin{array}{r} 4 \\ 29 \\ \times 75 \\ \hline 145 \end{array}$$

First multiply 5×29 .
Ignore the 7.

$$\begin{array}{r} 64 \\ 29 \\ \times 75 \\ \hline 145 \\ 2030 \end{array}$$

Then multiply 70×29 .
Ignore the 5. Since you are multiplying by 70, your answer will end **in a zero**. Start out by placing that zero in the ones place. Then, simply multiply 7×29 .

$$\begin{array}{r} 29 \\ \times 75 \\ \hline 145 \\ + 2030 \\ \hline 2175 \end{array}$$

Lastly add.
The answer is close to our estimate of 2100. That is good.

8. First estimate. Then multiply. Lastly check that your answer is reasonably close to your estimate.

| | | |
|--|--|--|
| <p>a. Estimate:</p> <p>_____</p> $\begin{array}{r} 93 \\ \times 27 \\ \hline \end{array}$ | <p>b. Estimate:</p> <p>_____</p> $\begin{array}{r} 55 \\ \times 46 \\ \hline \end{array}$ | <p>c. Estimate:</p> <p>_____</p> $\begin{array}{r} 87 \\ \times 16 \\ \hline \end{array}$ |
| <p>d. Estimate:</p> <p>_____</p> $\begin{array}{r} 61 \\ \times 90 \\ \hline \end{array}$ | <p>e. Estimate:</p> <p>_____</p> $\begin{array}{r} 24 \\ \times 18 \\ \hline \end{array}$ | <p>f. Estimate:</p> <p>_____</p> $\begin{array}{r} 98 \\ \times 51 \\ \hline \end{array}$ |

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5. Find a number to fit in the box so the equation is true.

| | | |
|---------------------------------------|--|--|
| a. $25 = 7 + \square \times 2$ | b. $72 \div 8 = (6 - 3) \times \square$ | c. $(4 + \square) \div 3 = 2 + 2$ |
|---------------------------------------|--|--|

6. Write an expression *or* an equation to match each written sentence. You do not have to solve.

| | |
|--|---|
| a. The difference of x and 9 | b. The sum of y and 3 and 8 equals 28. |
| c. The quotient of 60 and b is equal to 12. | d. The product of 8, c and d . |

7. Which expression matches the problem? Also, solve the problem.

| | | | | | |
|--|--|----------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| <p>Three girls divided equally the cost of buying four sandwiches for \$3.75 each. How much did each girl pay?</p> | <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; padding: 5px;">(1) $3 \times \\$3.75 - 4$</td> <td style="width: 50%; padding: 5px;">(2) $3 \times \\$3.75 \div 4$</td> </tr> <tr> <td style="padding: 5px;">(3) $\\$3.75 \div 4 \times 3$</td> <td style="padding: 5px;">(4) $4 \times \\$3.75 \div 3$</td> </tr> </table> | (1) $3 \times \$3.75 - 4$ | (2) $3 \times \$3.75 \div 4$ | (3) $\$3.75 \div 4 \times 3$ | (4) $4 \times \$3.75 \div 3$ |
| (1) $3 \times \$3.75 - 4$ | (2) $3 \times \$3.75 \div 4$ | | | | |
| (3) $\$3.75 \div 4 \times 3$ | (4) $4 \times \$3.75 \div 3$ | | | | |

8. Write a *single* expression (number sentence) for each problem, and solve.

| |
|--|
| <p>a. Bonnie and Ben bought an umbrella for \$12 and boots for \$17, and divided the cost equally. How much did each one pay?</p> |
| <p>b. Henry bought five cartons of milk for \$4.50 each. The grocer gave him \$2 off the total cost. How much did Henry pay?</p> |

9. Draw a bar model to represent the equations. Then solve them.

a. $R \div 4 = 544$

b. $4 \times R = 300$

10. Mark an “x” if the number is divisible by 2, 3, 5, 6, or 9.

| Divisible by | 2 | 3 | 5 | 6 | 9 |
|--------------|---|---|---|---|---|
| 534 | | | | | |
| 123 | | | | | |

| Divisible by | 2 | 3 | 5 | 6 | 9 |
|--------------|---|---|---|---|---|
| 1605 | | | | | |
| 2999 | | | | | |

11. Factor the following numbers to their prime factors.

a. 21
/\

b. 12
/\

c. 38
/\

d. 75
/\

e. 124
/\

f. 89
/\

Chapter 2: Large Numbers and the Calculator

Introduction

In this chapter, we study large numbers and place value up to billions—that is, up to 12-digit numbers. Students will also add, subtract, and round large numbers, and learn about exponents and powers.

Concerning exponents and powers, the focus is on powers of ten (such as 10^2 , 10^5 , 10^8 , and so on), which is what the student should master in this grade level. If your student has difficulties with exponents in general, there is no need to worry. Exponents and powers are taught from the very basics again in Math Mammoth grade 6.

Our number system is based on number 10, and it is *positional*: the place (location) of each digit matters in determining its value. Students have already learned quite a bit about place value. In this chapter, they will solidify their understanding of it. In particular, we examine multiplying numbers by powers of ten using a place value chart, and see how the common shortcut of tagging zeros to the end of a number actually has to do with the digits of the number *shifting* within the place value chart.

In this chapter, students will be introduced to the calculator for the first time, and therefore they will need a simple calculator (preferably a physical one). Some exercises may require the student to use a calculator on a computer or a phone, so as to fit more digits.

I have delayed the use of a calculator (as compared to many other maths curricula) for a good reason. I have received numerous comments on the harm that indiscriminate calculator usage can cause. If children are allowed to use calculators freely, their minds get “lazy,” and they will start relying on calculators even for simple things such as 6×7 or $320 + 50$. It is just human nature!

As a result, students may enter college without even knowing their multiplication tables by heart. Then they have trouble if they are required to use mental maths to solve simple problems.

Therefore, we educators need to *limit* calculator usage until the students are much older. Children *cannot* decide this for themselves, and definitely not in fifth grade.

However, I realise that the calculator is very useful, and students do need to learn to use it. In this curriculum, I try to not only show the students how to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practise estimating the result before using a calculator to find the exact answer, and choosing whether mental maths or a calculator is the best “tool” for the calculation.

From now on, the curriculum will show a little calculator symbol next to the exercises where I feel calculator usage is appropriate.

Pacing Suggestion for Chapter 2

This table does not include the chapter test as it is found in a different book (or file). Please add one day to the pacing for the test if you use it.

| The Lessons in Chapter 2 | page | span | suggested pacing | your pacing |
|--|------|---------|------------------|-------------|
| A Little Bit of Millions | 77 | 3 pages | 1 day | |
| Exponents and Powers | 80 | 3 pages | 1 day | |
| The Place Value System | 83 | 3 pages | 1 day | |
| Multiplying Numbers by Powers of Ten | 86 | 5 pages | 2 days | |

| The Lessons in Chapter 2 | page | span | suggested pacing | your pacing |
|--|------|----------|------------------|-------------|
| Adding and Subtracting Large Numbers | 91 | 3 pages | 1 day | |
| Rounding | 94 | 3 pages | 2 days | |
| The Calculator | 97 | 3 pages | 1 day | |
| When to Use the Calculator | 100 | 2 pages | 1 day | |
| Mixed Revision Chapter 2 | 102 | 2 pages | 1 day | |
| Chapter 2 Revision | 104 | 3 pages | 1 day | |
| Chapter 2 Test (optional) | | | | |
| TOTALS | | 30 pages | 12 days | |

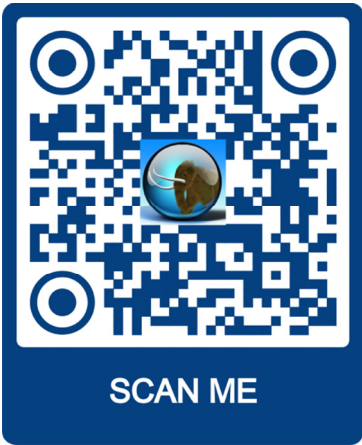
Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

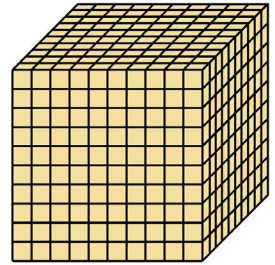
We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

<https://l.mathmammoth.com/gr5ch2>



A Little Bit of Millions

| | | |
|--|--|--|
| <p>If you count by whole thousands... (read the numbers aloud)</p> <p style="text-align: right;"> 995 000 996 000 997 000 998 000 999 000 </p> <p style="text-align: right;">...what comes after 999 thousand?</p> | <p>1 000 000</p> <p>A thousand thousands! It is called ONE MILLION.</p> | |
| <p>How big is one million? You've seen a cube like this to illustrate one thousand. Now imagine that <i>each little cube</i> in it was a 1000-cube in itself.</p> <p>It's a lot! It is 1000×1000 — a thousand copies of one thousand.</p> <p>A comma separates the millions places (digits) from the rest. After the millions, the rest of the number is read just like you have learned before.</p> | | |
| <p>3 47 5 000 000</p> <p>347 million 500 thousand</p> | <p>19 020 000</p> <p>19 million 20 thousand</p> | <p>5 040 326</p> <p>5 million 40 thousand 326</p> |



1. Continue the skip-counting patterns until you reach **one million**.

| | | | |
|--|--|--|--|
| <p>a.</p> <p>500 000</p> <p>600 000</p> | <p>b.</p> <p>940 000</p> <p>950 000</p> | <p>c.</p> <p>999 600</p> <p>999 700</p> | <p>d.</p> <p>999 994</p> <p>999 995</p> |
|--|--|--|--|

2. Write the numbers.

a. 18 million

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

b. 906 million

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

c. 2 million 400 thousand

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

d. 70 million 90 thousand

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

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The Place Value System

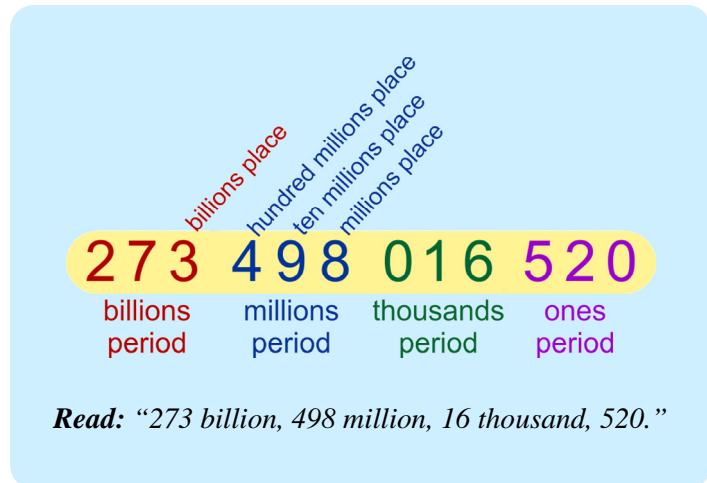
The number system we use is based on number ten, and it is a *positional* number system. This means that the position or **place** of each digit has to do with its value.

The **place** of a digit is its **location** within the number.

For example, in the number on the right, the digit 9 is in the ten millions place, and the digit 4 is in the hundred millions place.

In what place is digit 5? Digit 7?

Notice how the number ten has to do with these *places*. You see powers of ten at work! That is why our number system is a *base ten* system.



We group the digits of large numbers into groups of three. These groupings are called "periods," and they make it easy to read large numbers. Simply read each three digits as if it were a *number by itself*, and at the spaces, say the word "billion," "million," and "thousand."

1. A thousand thousands makes a million. What about a thousand millions? What do we call it? Also, write this number. Write it also using an exponent.

2. Arrange the digits of each number into groups of three with spaces. Then read each number.

a. 39204848486

b. 490255549632

c. 2843729584

d. 45038300820

e. 9000004000

f. 915008360000

3. Write the numbers. You will need to use zeros; be careful!

a. 159 billion 372 million 932 thousand 2 =

b. 7 billion 372 million 40 thousand 20 =

c. 57 billion 430 million 200 =

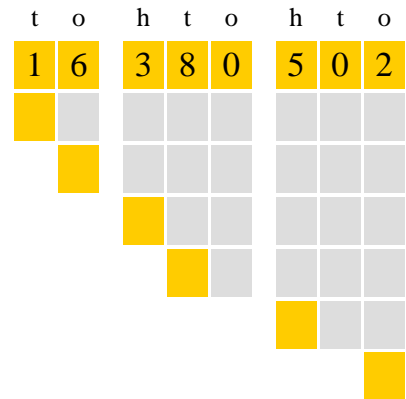
d. 607 billion 43 thousand 17 =

e. 372 million 150 =

f. 4 billion 901 thousand =

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|--|---|---|---|---|---|--|--|--|---|---|---|---|--|--|--|--|---|---|---|--|--|--|--|--|---|---|
| <p>What is the value of a digit?</p> <p>In the base ten system, each digit is multiplied by a certain power of ten, and this is its value.</p> <p>This power of ten comes from the <i>place</i> of the digit.</p> <p>For example, in 3 065 820, the digit 6 is in the <i>ten thousands</i> place. Its value is 6 times <i>ten thousand</i>, or 60 000.</p> <p>Here's a trick: If you set all the <i>other</i> digits in the number to zero, you will see the digit's value. See the chart.</p> | <table style="margin-left: auto; margin-right: auto; text-align: center;"> <tr> <td></td><td>o</td><td>h</td><td>t</td><td>o</td><td>h</td><td>t</td><td>o</td> </tr> <tr> <td></td><td>3</td><td>0</td><td>6</td><td>5</td><td>8</td><td>2</td><td>0</td> </tr> </table> <p>The value of the digit 3 is <table style="display: inline-table; text-align: center;"><tr><td>3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></p> <p>The value of the digit 6 is <table style="display: inline-table; text-align: center;"><tr><td></td><td></td><td>6</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table></p> <p>The value of the digit 5 is <table style="display: inline-table; text-align: center;"><tr><td></td><td></td><td></td><td>5</td><td>0</td><td>0</td><td>0</td></tr></table></p> <p>The value of the digit 8 is <table style="display: inline-table; text-align: center;"><tr><td></td><td></td><td></td><td></td><td>8</td><td>0</td><td>0</td></tr></table></p> <p>The value of the digit 2 is <table style="display: inline-table; text-align: center;"><tr><td></td><td></td><td></td><td></td><td></td><td>2</td><td>0</td></tr></table></p> <p style="text-align: right;"><i>If you add all these, ↑ you will get the number itself!</i></p> <p style="text-align: center;">(The “h”, “t”, “o” refer to hundreds, tens, and ones.)</p> | | o | h | t | o | h | t | o | | 3 | 0 | 6 | 5 | 8 | 2 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | | | 6 | 0 | 0 | 0 | 0 | | | | 5 | 0 | 0 | 0 | | | | | 8 | 0 | 0 | | | | | | 2 | 0 |
| | o | h | t | o | h | t | o | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | 0 | 6 | 5 | 8 | 2 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 6 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 5 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | 8 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | 2 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

4. Consider the number 16 380 502. Use the chart to help you if you'd like. In that number...



- a. What is the value of digit 5?
- b. What is the value of digit 8?
- c. What is the value of digit 1?

5. In what place is the underlined digit? What is its value? You can use the charts to help you.

| | |
|--|--|
| <p>a. 293 <u>4</u>76 020</p> <p>Place: _____</p> <p>Value: _____</p> | <p>b. 3 29<u>9</u> 005 392</p> <p>Place: _____</p> <p>Value: _____</p> |
| <p>c. <u>2</u>8 837 402 000</p> <p>Place: _____</p> <p>Value: _____</p> | <p>d. 2<u>9</u>3 476 020</p> <p>Place: _____</p> <p>Value: _____</p> |
| <p>e. 3 <u>2</u>99 005 392</p> <p>Place: _____</p> <p>Value: _____</p> | <p>f. 28 837 4<u>3</u>2 000</p> <p>Place: _____</p> <p>Value: _____</p> |



To write a number **in expanded form** means we write it as a sum of its parts, according to place value. We take each digit and multiply it by the power of ten that corresponds to the place of the digit, and lastly add it all up. See the examples. The power of ten can be written out as a normal number, or written with an exponent.

Example 1. In expanded form, $302\,478 = 3 \times 100\,000 + 2 \times 1000 + 4 \times 100 + 7 \times 10 + 8 \times 1$

Example 2. In expanded form, $6\,090\,507 = 6 \times 10^6 + 9 \times 10^4 + 5 \times 10^2 + 7 \times 10^0$

Note that 10^0 (10 to the 0th power) indeed equals 1!

6. Fill in.

a. $269\,115 = \square \times 100\,000 + \square \times 10\,000 + \square \times 1000 + \square \times 100 + \square \times 10 + \square \times 1$

b. $6\,087\,240 = \square \times 10^6 + 8 \times 10^{\square} + 7 \times 10^{\square} + \square \times 10^2 + \square \times 10^1$

7. Write the numbers in expanded form. You can choose whether to write the powers of ten with exponents or as normal numbers.

a. $87\,034 =$

b. $2\,167\,900 =$

c. $97\,225 =$

d. $708\,340 =$

8. (Optional) The next groupings of digits (periods) after billions are called **trillions**, **quadrillions**, and **quintillions**.

a. Read the number 34 506 039 566 392 000

b. Write: 7 quintillion, 204 quadrillion, 9 billion, 43 million

Mystery Number

(1)

“In my hundred millions place there’s a 7. I only have one ‘2’, and its value is 2000. My millions digit is double the thousands digit, and the hundreds digit is double the millions digit. The rest are zeros.”

I am _____

(2)

“I only have odd numbers and zeros as digits — each odd number used once. The value of one of them is 10 000. In the hundred thousands place I have an odd digit that is *not* a prime. My millions digit is that digit divided by 3. And the other two odd number digits are in the ten millions place and in the tens place — the smaller one is in the bigger place.”

I am _____

Multiplying Numbers by Powers of Ten

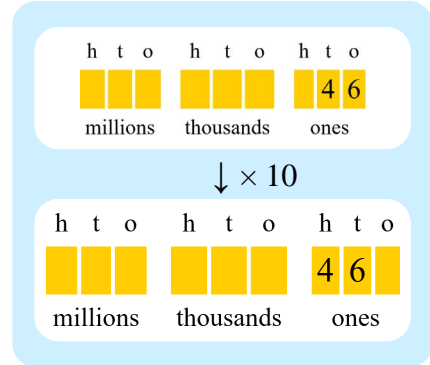
Let's consider in detail what actually happens when we multiply a number by a power of ten.

Example 1. We will study 10 times 46, and consider 46 as **4 tens and 6 ones**, according to place value.

When 6 ones are multiplied by 10, they become 6 tens.
When 4 tens are multiplied by 10, they become 4 hundreds.

This means **the digits shift** one place to the left, on the place value chart. (Why?)

Lastly, we simply add a zero to the ones place as a placeholder, as is customary, and write our answer as 460.



In reality, ALL the places that don't have a digit from 1 to 9, have a zero! So, our number could be written as 00460 or 0000460. But it is customary to omit the leading zeros in whole numbers.

Stop and think about this: the number "four hundreds and six tens" *could* be written as 46__ or 46* or some other way that would still indicate that 4 is for hundreds and 6 is for tens — or indicate *the position* of those digits. Zero is just a placeholder for that purpose!

1. What happens to 72 when you multiply it by 10 repeatedly? Use the place value charts to help.

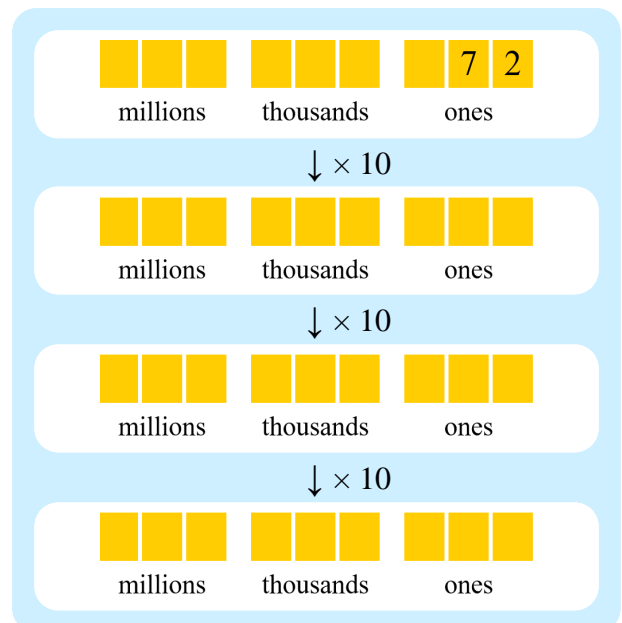
$$10 \times 72 = \underline{\hspace{2cm}}$$

$$10 \times 10 \times 72 = \underline{\hspace{2cm}}$$

$$10 \times 10 \times 10 \times 72 = \underline{\hspace{2cm}}$$

$$10 \times 10 \times 10 \times 10 \times 72 = \underline{\hspace{2cm}}$$

What do you notice?



Continue the pattern for two more steps. (Using exponents will help.)

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Chapter 3: Problem Solving

Introduction

We start out this chapter by studying simple equations, presented as pan balance puzzles. The pan balance works very well for modelling the process of solving equations. In the second lesson, students use the bar model to help them solve equations. The equations on this level are very simple. More complex equations are presented in grade 6 and especially in grade 7 (pre-algebra).

The bulk of this chapter is then spent on the topic of problem solving, focusing on problems that involve a fractional part of a whole in some manner.

These lessons teach students to draw a visual bar model for the problems. The bar model is a very good tool to help students conceptualise and solve problems that otherwise they might need an algebraic equation for. At the same time, using the bar model helps the students develop algebraic thinking. Essentially, one block in the bar model corresponds to the unknown x in an equation.

Encourage students to plan a solution for a problem before starting the solution, instead of simply jumping in without much thinking. Also, the problems in these lessons give a good opportunity to teach students to check their final answer: does it make sense? Does it fit with what the problem states?

Many students are afraid of word problems. That doesn't have to be. One key is to get students used to solving problems and allow them enough practice at the right difficulty level. Another important factor is that we educators don't "chastise" students for errors or put down errors. Just the opposite — an error should be seen as a great opportunity for learning. In fact, brain research has proven that our brains grow and make new connections when we think about a mistake we made!

When a student has made a mistake, you can ask, "Can you show me how you got your answer?", and not even say there was a mistake. As they explain their thought process, you can help them, or they might notice the error themselves.

Pacing Suggestion for Chapter 3

This table does not include the chapter test as it is found in a different book (or file). Please add one day to the pacing for the test if you use it.

| The Lessons in Chapter 3 | page | span | suggested pacing | your pacing |
|--|------|----------|---------------------|----------------|
| Balance Problems and Equations, Part 1 | 109 | 3 pages | 1 day | |
| Balance Problems and Equations, Part 2 | 112 | 3 pages | 1 day | |
| Problem Solving with Bar Models 1 | 115 | 3 pages | 1 day | |
| Problem Solving with Bar Models 2 | 118 | 2 pages | 1 day | |
| Problem Solving with Bar Models 3 | 120 | 2 pages | 1 day | |
| Problem Solving with Bar Models 4 | 122 | 2 pages | 1 day | |
| Miscellaneous Problems | 124 | 2 pages | 1 day | |
| Mixed Revision Chapter 3 | 126 | 2 pages | 1 day | |
| Chapter 3 Revision | 128 | 3 pages | 1 day | |
| Chapter 3 Test (optional) | | | | |
| TOTALS | | 22 pages | 9 days | |

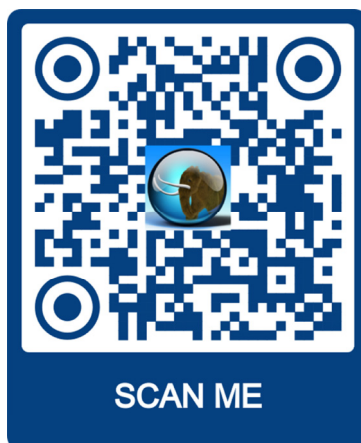
Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

<https://l.mathmammoth.com/gr5ch3>



Balance Problems and Equations 1

Here you see a pan balance, or scales, and some things on both pans. Each rectangle represents an unknown (and “weighs” the same, or has the same value).

Since the balance is *balanced* (neither pan is going down—they are level with each other), the two sides (pans) of the scales weigh the same.

This portrays a mathematical equation: what is in the left pan equals what is in the right pan. (Things in the same pan are simply added.)

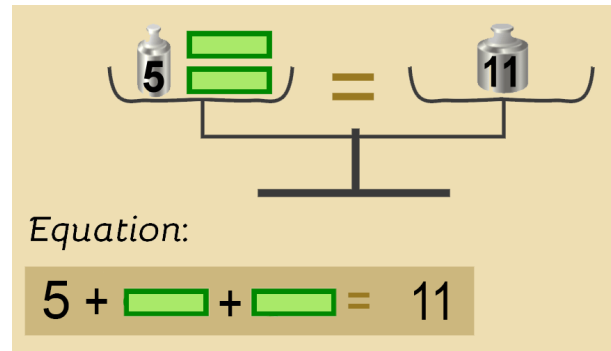
The equation is:

$$5 + \square + \square = 11$$

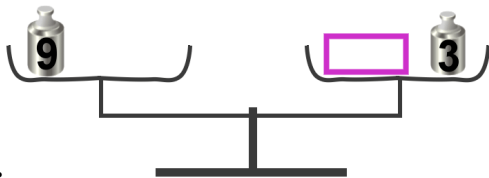
(If it helps you, you can think of kilograms.)

When we figure out how much the unknown shape weighs, we solve the equation.

The solution is: $\square = 3$

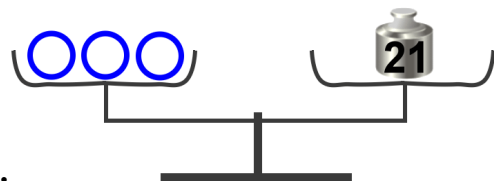


1. Write an equation for each balance. Then use mental maths to solve how much each geometric shape “weighs.” You can write a number inside each of the geometric shapes to help you.



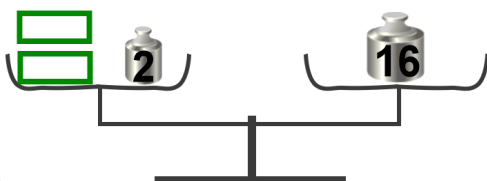
Equation: $9 = \square + 3$

Solution: $\square = 6$



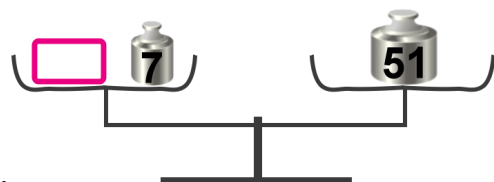
Equation: _____

Solution: $\bigcirc = \underline{\hspace{2cm}}$



Equation: _____

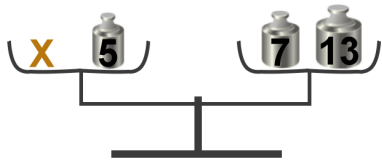
Solution: $\square = \underline{\hspace{2cm}}$



Equation: _____

Solution: $\square = \underline{\hspace{2cm}}$

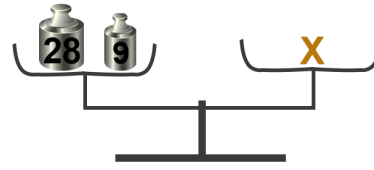
From now on we will use x for the unknown instead of a geometric shape. It is the most commonly used letter of the alphabet to signify an unknown.



$$\begin{aligned} x + 5 &= 7 + 13 \\ x + 5 &= 20 \\ x &= 15 \end{aligned}$$

Example 1. To solve this equation, first add 7 and 13 that are in the right “pan”.

We get $x + 5 = 20$. The solution is easy to see now with mental maths: $x = 15$. You can also use subtraction: $x = 20 - 5 = 15$.



$$\begin{aligned} 28 + 9 &= x \\ 37 &= x \\ x &= 37 \end{aligned}$$

Example 2. Sometimes x is on the right side of the equation. That is not a problem. In the last step you can flip the sides, so that your last line will be $x = (\text{something})$.

Notice that we *align the equal signs* when solving an equation. It keeps everything tidy and easy to read.

2. Write an equation. Write a second step if necessary. Lastly write what x stands for.

a.

$$\begin{aligned} \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ x &= \underline{\hspace{2cm}} \end{aligned}$$

b.

$$\begin{aligned} \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ x &= \underline{\hspace{2cm}} \end{aligned}$$

3. Draw x 's and weights on the left and right sides on the two pans to match the given equation, then solve. You may not need all the empty lines provided.

a.

$$\begin{aligned} x + 18 &= 5 + 31 \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \end{aligned}$$

b.

$$\begin{aligned} 8 + 17 &= 11 + x \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} &= \underline{\hspace{2cm}} \end{aligned}$$

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Divide Decimals by Whole Numbers 2

Sometimes we can divide a decimal by a whole number using mental maths.

1. One way is to **think of multiplication “backwards.”**

To solve $4.5 \div 5$, think: “*What number multiplied by 5 will give me 4.5?*”

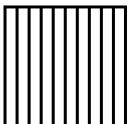
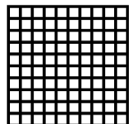
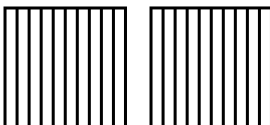
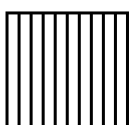
In other words, $\underline{\quad} \times 5 = 4.5$.

The answer is 0.9.

2. Another way is this. **Think of sharing evenly** the tenths, hundredths, or thousandths among some people.

For example, $0.035 \div 5$ is “*35 thousandths divided by 5*”. This is the same idea as dividing 35 *apples* by 5. The answer is 7, but since our items are thousandths, the answer is 7 *thousandths*, or 0.007.

1. First shade the parts. Then divide and write a division sentence.

| | |
|--|---|
|  <p>a. Colour in 0.3. Divide it into 3 parts.</p> <p>$\underline{\quad} \div \underline{3} = \underline{\quad}$</p> |  <p>b. Colour in 0.64. Divide it into 2 parts.</p> <p>$\underline{\quad} \div \underline{\quad} = \underline{\quad}$</p> |
|  <p>c. Colour in 1.8. Divide it into 3 parts.</p> <p>$\underline{\quad} \div \underline{\quad} = \underline{\quad}$</p> |  <p>d. Colour in 0.1. Divide it into 10 parts.</p> <p>$\underline{\quad} \div \underline{\quad} = \underline{\quad}$</p> |

2. Write the division problems with numbers, and solve. Think of equal sharing. Remember also that you can check each division by multiplication.

a. 9 tenths divided by 3 equals ... $\underline{\quad} \div \underline{\quad} = \underline{\quad}$

b. 72 thousandths divided by 9 equals ... $\underline{\quad} \div \underline{\quad} = \underline{\quad}$

c. 54 hundredths divided by 6 equals ... $\underline{\quad} \div \underline{\quad} = \underline{\quad}$

d. 122 hundredths divided by 2 equals ... $\underline{\quad} \div \underline{\quad} = \underline{\quad}$

3. Divide mentally.

| | | |
|--|--|---|
| a. $0.024 \div 6 = \underline{\quad}$ | d. $0.49 \div 7 = \underline{\quad}$ | g. $5.40 \div 9 = \underline{\quad}$ |
| b. $0.24 \div 6 = \underline{\quad}$ | e. $1.2 \div 3 = \underline{\quad}$ | h. $0.20 \div 4 = \underline{\quad}$ |
| c. $2.4 \div 6 = \underline{\quad}$ | f. $0.056 \div 7 = \underline{\quad}$ | i. $0.050 \div 10 = \underline{\quad}$ |

Chapter 5: Graphing

Introduction

This chapter introduces the coordinate grid, but only in the first quadrant. Students learn to plot points and to read their coordinates. They practise using grids with different scaling, and also draw shapes and lines.

Then, students study simple number patterns (number rules), and plot points produced by the rule. This concept will later on lead to the study of *functions* (in 8th grade and beyond).

Next, we study line graphs, which is a natural application of the coordinate grid. Students read and make line graphs, including double line graphs, and answer questions about data already plotted.

At the end of the chapter, we also revise the concept of average (also called the *mean*), and see how it relates to line graphs.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file). Please add one day to the pacing for the test if you use it.

| The Lessons in Chapter 5 | page | span | suggested pacing | your pacing |
|--|------|-----------------|---------------------|----------------|
| The Coordinate Grid | 175 | <i>3 pages</i> | 1 day | |
| The Coordinate Grid, Part 2 | 178 | <i>2 pages</i> | 1 day | |
| Number Patterns in the Coordinate Grid | 180 | <i>3 pages</i> | 1 day | |
| More Number Patterns in the Coordinate Grid | 183 | <i>3 pages</i> | 1 day | |
| Line Graphs | 186 | <i>4 pages</i> | 2 days | |
| Double and Triple Line Graphs | 190 | <i>3 pages</i> | 1 day | |
| Average (Mean) | 193 | <i>3 pages</i> | 1 day | |
| | | | | |
| Mixed Revision Chapter 5 | 196 | <i>3 pages</i> | 2 days | |
| Chapter 5 Revision | 199 | <i>2 pages</i> | 1 day | |
| Chapter 5 Test (optional) | | | | |
| TOTALS | | <i>26 pages</i> | 11 days | |

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<https://l.mathmammoth.com/gr5ch5>



The Coordinate Grid

This is a **coordinate grid**. It consists of two number lines that are set perpendicular (at right angles) to each other.

The horizontal number line is called the **x-axis**. The vertical one is called the **y-axis**.

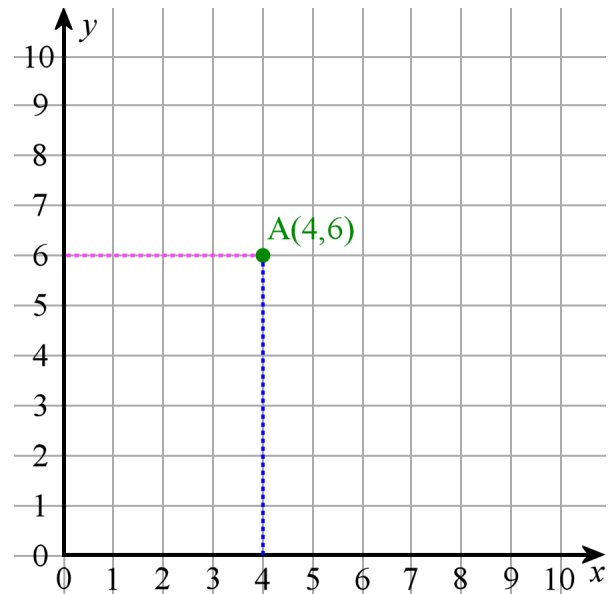
You can see one point, called “A,” that is drawn or *plotted* on the grid.

Since we have two number lines, we use *two* numbers (4 and 6) to signify its location. Those numbers are the **coordinates** of the point A.

The first number, 4, is the **x-coordinate** of the point A. It is called the *x*-coordinate because point A is four units from zero in the horizontal direction (direction of the *x*-axis).

We can see that by drawing a straight line down from A. The line *intersects*, or “hits,” the *x*-axis at 4.

The second number is the **y-coordinate** of the point A. In the vertical direction, point A is six units from zero. When we draw a line directly towards left from A, it intersects the *y*-axis at 6.



We write the two coordinates of a point inside parentheses, separated by a comma: (4, 6).

Note: (4, 6) is an **ordered pair**: the order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is always the *y*-coordinate, not vice versa.

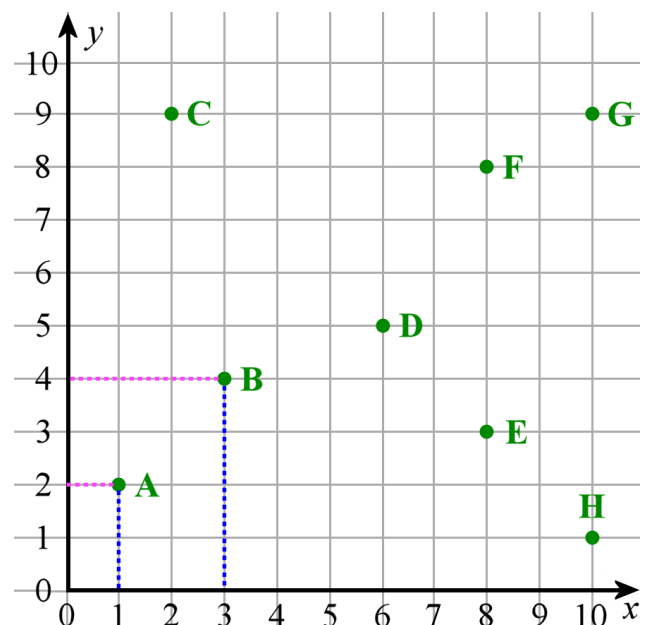
1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in. (The helping lines are not necessary to draw; they are just that — *helping* lines. You can draw them if they help you.)

A (__, __) B (__, __)

C (__, __) D (__, __)

E (__, __) F (__, __)

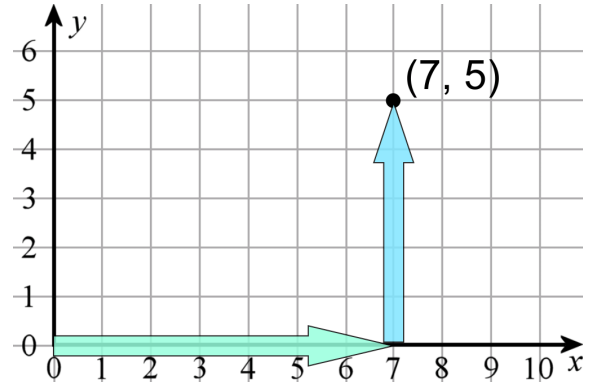
G (__, __) H (__, __)



To plot points, you can first “travel” on the x -axis from the point $(0, 0)$ (the **origin**) the number of units indicated by the x -coordinate.

Then travel UP as many units as the y -coordinate indicates.

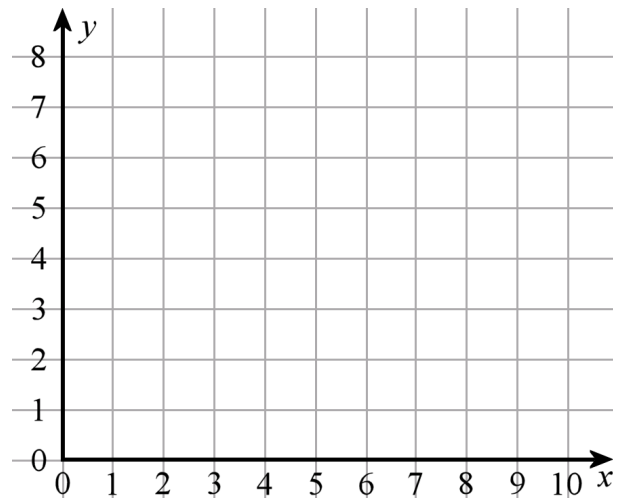
The image shows an example of how to plot $(7, 5)$.



2. Plot the following points on the coordinate grid. Then join them with line segments in the alphabetical order. What do you get?

A(1, 5) B(4, 3) C(4, 6)

D(7, 5) E(6, 8)

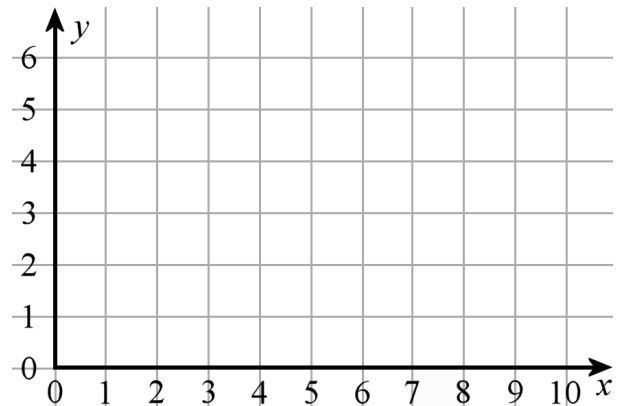


3. **Zero as a coordinate.** Plot the following points in the grid on the right.

A(0, 6) B(0, 3) C(0, 0)

D(5, 0) E(9, 0)

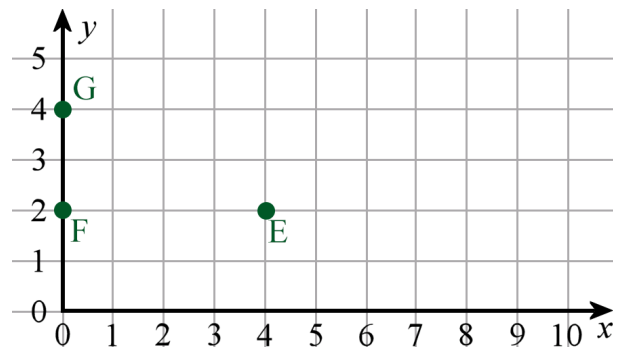
What do you notice?



4. **a.** Write the coordinates of the points E, F, and G.

b. Plot a fourth point, H, so that when you join E, F, G, and H with line segments, you will get a rectangle.

c. What are the coordinates of H?



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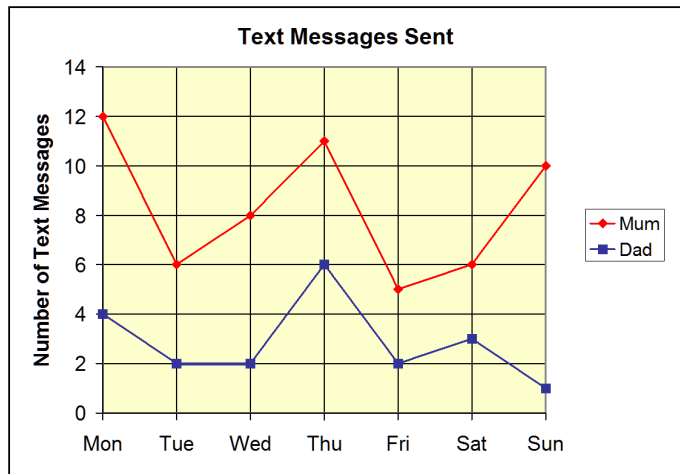
Double and Triple Line Graphs

A double-line graph has two lines, for two different sets of data.

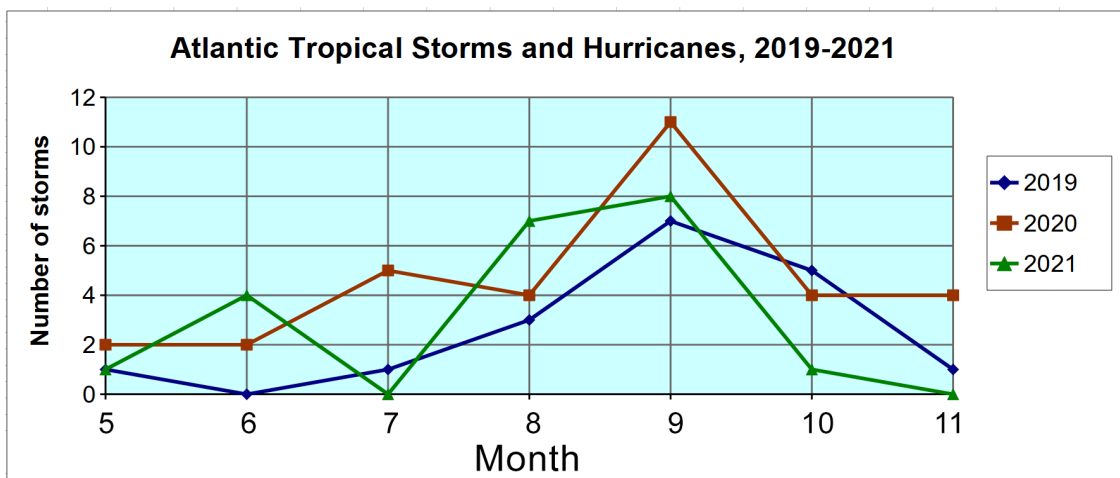
We can distinguish these lines by marking the data points in different manners (e.g. with circles and triangles) and by using different colours.

Usually double-line graphs have a *legend* that explains which line belongs to which data set.

The same principles apply to line graphs with even more data sets (more lines).



- Refer to the double-line graph above about the text messages Mum and Dad sent.
 - How many more messages did Mum send on Thursday than Dad?
 - Find the day with the *greatest difference* between the number of messages Mum sent and the number of messages Dad sent.
 - Find the day with the *least difference* between the number of messages Mum sent and the number of messages Dad sent.
- The graph shows the number of tropical storms and hurricanes in three Atlantic hurricane seasons.



- Which year had the most storms?
- Based on this graph, which month is the most active month?
- Which is the second most active month?

3. (An optional project) Go to Wikipedia and find out how many hurricanes and tropical storms there were for each month from May through November, for some particular years. Count a storm that extends into two months by the month it started.

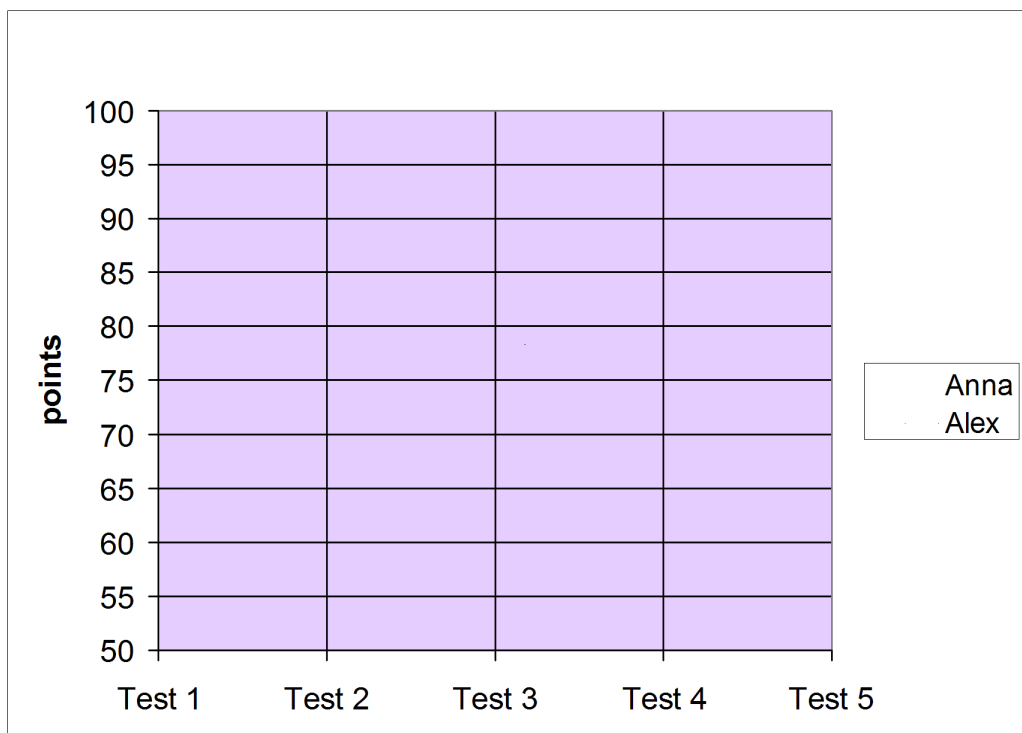
Add that data to the line graph on the previous page, or make a new one.

Compare the hurricane seasons of the various years. Is a particular year much more active or much less active than others?

4. The table shows Anna's and Alex's test scores in five science tests.

- a. Draw a double-line graph of the data. Choose two different markers for the two data sets, and add a legend based on that. Add a title, also.

| | Anna | Alex |
|--------|------|------|
| Test 1 | 65 | 72 |
| Test 2 | 62 | 66 |
| Test 3 | 77 | 71 |
| Test 4 | 85 | 59 |
| Test 5 | 82 | 68 |

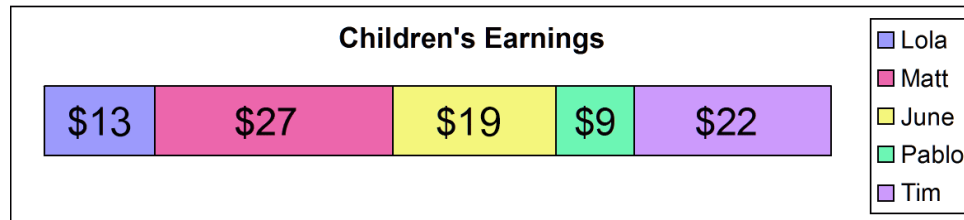


- b. Describe Anna's performance over time.
(Did she improve? Get worse? Stay about the same?)
- c. In which test was the difference between Anna's and Alex's point count the greatest?

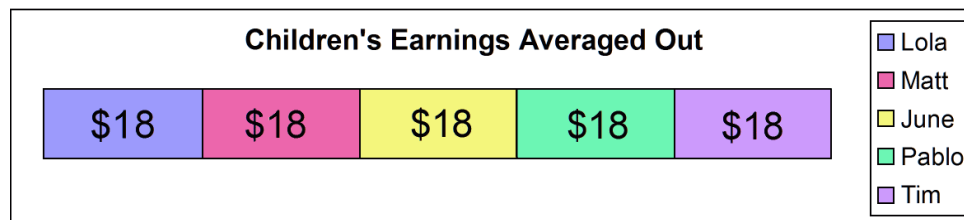
In which test was it the smallest?

Average (Mean)

Example 1. Five children earned these amounts of money for a job: \$13, \$27, \$19, \$9, and \$22. The graph below shows visually how much each child earned.



Together, they earned \$90. If this \$90 had been divided equally among the children, each child would have gotten \$18. (It was not, because the children got paid according to how much they worked.) This \$18 is the **average** pay.



The graph above shows the situation *if* each child had received the average earning (\$18). Notice that \$18 is sort of in the “middle” or in between the lowest and highest earnings.

- To calculate the average, first add all the numbers in the data set, and then divide the sum by the number of items in the data set. In other words,

$$\text{average} = \frac{\text{sum of all the items}}{\text{the number of items}}$$

- The average is always somewhere in the middle of a set of data: it is more than the smallest number and less than the largest number of the data.
- The average is also called the **mean**. We will use both terms in this lesson so you get used to both.

1. Calculate the average of the data sets. Do not use a calculator.

a. 2, 4, 5, 9, 0, 4, 1, 7

b. 13, 16, 20, 22, 16, 13, 17, 12, 15

2. Calculate the mean of the data sets to the nearest tenth. This time use a calculator.

a. 2, 4.3, 5, 9, 4.7, 9.4, 3.7, 5.1

b. 312, 288, 284, 329, 293, 302

