19. Use substitution to solve: $\begin{cases} 3x + 5y = 4 \\ 10x - 15y = -50 \end{cases}$

20. Graph: $\begin{cases} x - 3y \geq 6 \\ x \geq -3 \end{cases}$

Solve by factoring. Rearrange if necessary.

21. $-5x^2 - 2x + 3x^3 = 0$

22. $-10x - 4 + 6x^2 = 0$

23. $8x + 4 + 3x^2 = 0$

24. $24x^2 + 9x^3 + 12x = 0$

25. $2p^2 - 3p - 5 = 0$

26. $8 + 18x + 4x^2 = 0$

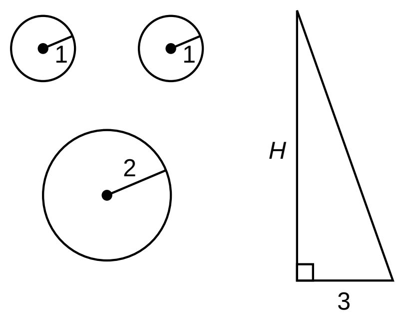
Simplify:

27. $\dfrac{x^a\left(x^{a/2+4}\right)^2 y^b}{y^{b/3} x^{a/6}}$

28. $\dfrac{2 - 3i^3}{i + 2i^2 + 3i^3}$

29. $\dfrac{4 + 2\sqrt{5}}{5 - 3\sqrt{5}}$

30. The radii of the circles are 1 unit, 1 unit, and 2 units, as shown. The base of the triangle is 3 units long. The area of the triangle equals the sum of the areas of the three circles. What is the altitude of the triangle?



LESSON 106 More on systems of three equations

Some systems of three equations in three unknowns do not have all three variables in each of the equations. These equations can also be solved by using the substitution method or the elimination method.

example 106.1 Solve: $\begin{cases} 2x + 3y = -4 & \text{(a)} \\ x - 2z = -3 & \text{(b)} \\ 2y - z = -6 & \text{(c)} \end{cases}$

solution One variable is missing in each equation. We can see this better if we write the equations in expanded form.

$$\begin{array}{llcrcrcl} \text{(a)} & 2x & + & 3y & & & = & -4 \\ \text{(b)} & x & & & - & 2z & = & -3 \\ \text{(c)} & & & 2y & - & z & = & -6 \end{array}$$

The first step is to add any two of the equations so that one variable is eliminated. We could add (a) and (b) to eliminate x; or add (b) and (c) to eliminate z; or add (a) and (c) to eliminate y. We choose to eliminate x, so we add equation (a) to the product of equation (b) and (–2).

$$\begin{array}{rrcrcrcr}
\text{(a)} & 2x & + & 3y & & & = & -4 \\
(-2)\text{(b)} & -2x & & & + & 4z & = & 6 \\
\hline
\text{(d)} & & & 3y & + & 4z & = & 2
\end{array}$$

The resulting equation, (d), has y and z as variables. So does equation (c). We use equations (d) and (c) to eliminate z by adding equation (d) to the product of (4) and equation (c).

$$\begin{array}{rrcrcr}
(4)\text{(c)} & 8y & - & 4z & = & -24 \\
\text{(d)} & 3y & + & 4z & = & 2 \\
\hline
& 11y & & & = & -22
\end{array}$$

$$\mathbf{y = -2}$$

Now we replace y with -2 in equation (a) and solve for x. Then we replace y with -2 in equation (c) and solve for z.

$$\begin{array}{lrcl}
\text{(a)} & 2x + 3(-2) & = & -4 \\
& 2x - 6 & = & -4 \\
& 2x & = & 2 \\
& \mathbf{x} & \mathbf{=} & \mathbf{1}
\end{array}
\qquad
\begin{array}{lrcl}
\text{(c)} & 2(-2) - z & = & -6 \\
& -4 - z & = & -6 \\
& -z & = & -2 \\
& \mathbf{z} & \mathbf{=} & \mathbf{2}
\end{array}$$

Thus, our solution is the ordered triple **(1, –2, 2).**

example 106.2 Solve: $\begin{cases} 3y - 2z = -12 & \text{(a)} \\ 2x - 3z = -5 & \text{(b)} \\ x - 2y = 6 & \text{(c)} \end{cases}$

solution Although it is not necessary, we will begin by writing the equations in expanded form.

$$\begin{array}{lrcrcrcr}
\text{(a)} & & & 3y & - & 2z & = & -12 \\
\text{(b)} & 2x & & & - & 3z & = & -5 \\
\text{(c)} & x & - & 2y & & & = & 6
\end{array}$$

This time we decide to eliminate y, so we will add the product of 2 and equation (a) to the product of 3 and equation (c).

$$\begin{array}{rrcrcrcr}
(2)\text{(a)} & & & 6y & - & 4z & = & -24 \\
(3)\text{(c)} & 3x & - & 6y & & & = & 18 \\
\hline
\text{(d)} & 3x & & & - & 4z & = & -6
\end{array}$$

Now equation (b) also is an equation in x and z, so we decide to add the product of -3 and equation (b) to the product of 2 and equation (d).

$$\begin{array}{rrcrcr}
(-3)\text{(b)} & -6x & + & 9z & = & 15 \\
(2)\text{(d)} & 6x & - & 8z & = & -12 \\
\hline
& & & \mathbf{z} & \mathbf{=} & \mathbf{3}
\end{array}$$

Now we will replace z with 3 in equation (a) and (b) and solve for x and y.

$$\begin{array}{lrcl}
\text{(a)} & 3y - 2(3) & = & -12 \\
& 3y - 6 & = & -12 \\
& 3y & = & -6 \\
& \mathbf{y} & \mathbf{=} & \mathbf{-2}
\end{array}
\qquad
\begin{array}{lrcl}
\text{(b)} & 2x - 3(3) & = & -5 \\
& 2x - 9 & = & -5 \\
& 2x & = & 4 \\
& \mathbf{x} & \mathbf{=} & \mathbf{2}
\end{array}$$

So the solution to this system is the ordered triple **(2, –2, 3).**

It was not necessary to use elimination as the first step. For example, we could have solved equation (c) for x.

$$x = 6 + 2y$$

and substituted $6 + 2y$ for x in equation (b).

$$\begin{aligned} &\text{(b)} \quad 2(6 + 2y) - 3z = -5 \\ &\qquad\quad 12 + 4y - 3z = -5 \\ &\text{(e)} \qquad\qquad 4y - 3z = -17 \end{aligned}$$

Now equation (e) could be used with equation (a) to solve for y and z.

practice

Solve: $\begin{cases} 3x + 2y = 6 \\ 2x - z = 9 \\ 3y - z = 10 \end{cases}$

problem set 106

1. Jerry started a used parts establishment. He marked up the items 30 percent of the purchase price. If one item sold for \$715, what did Jerry pay for it?

2. The small plane could go 6 times as fast as the small car. Thus, the plane could go 1200 miles in only 1 hour less than it took the car to go 250 miles. Find the rate and the time of the small plane and the rate and the time of the small car.

3. The near-stagnant river flowed at only 2 miles per hour. Harold's boat could go 56 miles down the river in one-half the time it took to go 80 miles up the river. What was the speed of his boat in still water?

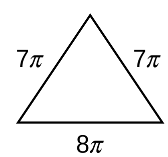
4. The temperature of a quantity of an ideal gas was held constant at 500°C. The initial pressure and volume were 700 torr and 500 ml. What would the final pressure be if the volume were increased to 1000 ml?

Solve:

5. $\begin{cases} 3x - 3y = 9 \\ 4x + z = 5 \\ 4y + 2z = -10 \end{cases}$

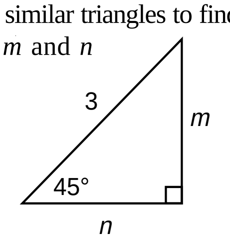
6. $\begin{cases} 2x - 2y - z = 9 \\ 3x + 3y - z = 6 \\ x + y + z = -2 \end{cases}$

7. Find the area of this isosceles triangle. Units are in centimeters.

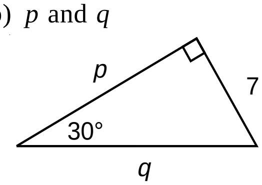


8. Use similar triangles to find:

(a) m and n



(b) p and q



Show that each number is a rational number by writing it as a quotient of integers:

9. $0.0007\overline{013}$

10. $4.10\overline{26}$

11. Complete the square as an aid in graphing: $y = -x^2 - 2x - 3$

12. Graph on a number line: $-|x| - 4 \ngtr 0$; $D = \{\text{Integers}\}$

13. Find the number that is $\frac{1}{10}$ of the way from $3\frac{1}{3}$ to $6\frac{1}{2}$.

Solve:

14. $\begin{cases} \frac{3}{5}x - \frac{1}{4}y = 5 \\ 0.012x + 0.07y = 2.20 \end{cases}$

15. $\begin{cases} 5x - y = 3 \\ xy = 4 \end{cases}$

16. $\begin{cases} x^2 + y^2 = 7 \\ 2x - y = 2 \end{cases}$

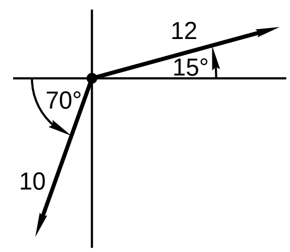
17. Use substitution to solve: $\begin{cases} 5x - 3y = 27 \\ 2x - 5y = 26 \end{cases}$

18. Graph: $\begin{cases} 3x - 4y \geq 8 \\ y > -2 \end{cases}$

19. Simplify: $\dfrac{3 - 2i^2 - i}{3i^3 + 3i + 2}$

20. Solve $3x^2 - x - 7 = 0$ by completing the square.

21. Find the resultant of the vectors shown.



22. Does this set of ordered pairs designate a function?

(4, 6), (–3, –2), (8, 5), (4, 6), (3, –2), (8, 5), (11, –3)

23. Find $\psi\theta\left(\frac{1}{2}\right)$ if $\psi(x) = x + 2$; $D = \{\text{Reals}\}$, and $\theta(x) = x^2$; $D = \{\text{Integers}\}$.

24. Divide $m^3 - p^3$ by $m - p$.

Solve by factoring. Rearrange as necessary. Always look for a common factor.

25. $3x^2 + 7x + 2 = 0$

26. $3x^2 + x - 2 = 0$

27. $2z^2 + 13z + 15 = 0$

28. $33p^2 + 45p + 6p^3 = 0$

29. $3p^2 - 13p - 10 = 0$

30. $-11a + 15 = -2a^2$

LESSON 107 Numbers, numerals, and value • Number word problems

107.A numbers, numerals, and value

We remember that a numeral is a single symbol or a meaningful arrangement of symbols that
we use to represent a particular number. We say that the value of each of the following numerals is three because each numeral represents the number 3.

$$3 \qquad 1 + 1 + 1 \qquad \frac{81}{27} \qquad 2^3 - 5 \qquad 2^2 - 1$$

Thus, we see that **number** and **value** mean the same thing. Also, we see that it would be redundant to speak of the value of a number because that is the same thing as saying the number of a number. But we can speak of the value of a numeral because this is the number represented by the numeral. Since paying excessive attention to the difference between a number and a numeral is often counterproductive, we sometimes use the word number when we should use the word numeral. See if you can find where this mistake is made in this lesson. The mistake is not serious.