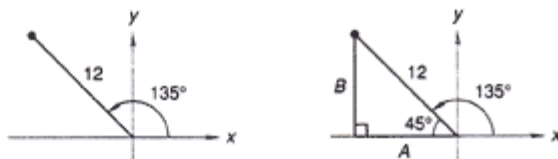## LESSON 27   Related Angles • Signs of Trigonometric Functions

27.A
related angles

When we draw the vector $12\underline{/135°}$, we measure the angle from the positive x axis, as we show in the left-hand figure.

In the right-hand figure, we complete the triangle by drawing a perpendicular from the end of the vector to the x axis. We can find the rectangular coordinates of the vector by solving for A and B.
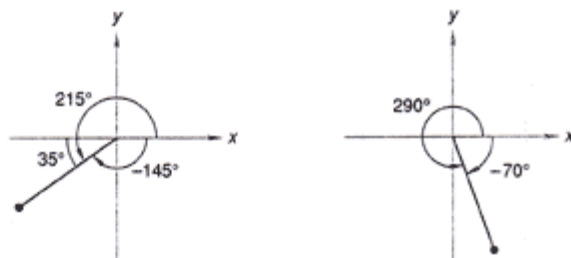
$$A = 12\cos 45° = 12(0.707) = 8.48$$

$$B = 12\sin 45° = 12(0.707) = 8.48$$

Thus, we have

$$12\underline{/135°} = -8.48\hat{i} + 8.48\hat{j}$$

In this solution, we used the angle 135° to help locate the vector. Then we used the 45° angle to solve the triangle. Many authors call the acute angle between the vector and the x axis the **related angle.** In this case, they would say that 45° is the related angle of 135°.

In the left-hand figure, we see that 35° is the related angle of both +215° and −145°. In the right-hand figure, we see that 70° is the related angle of both +290° and −70°. **The related angle is always a positive angle and is the acute angle between the vector and the x axis.**

27.B
signs of trigonometric functions

In the beginning of this lesson, we had a vector whose angle was 135°. To find the rectangular components of this vector, we used the related angle, which was 45°, and drew a triangle all sides of which were considered to be positive. We used the cosine of 45° and the sine of 45° to solve this triangle. We did not use the cosine of 135° or the sine of 135°. We will always use this procedure to find the components of vectors.

To discuss trigonometric functions, however, it is necessary to define trigonometric functions of angles that are not first-quadrant angles. **The absolute value of the trigonometric function of any angle is the same as that of the related angle, but the sign (+ or −) of the trigonometric function is determined by the quadrant in which the vector lies.** We determine the sign by considering the signs of the rectangular coordinates of the vector. **The hypotenuse is the length of the vector and is always considered to be positive.**