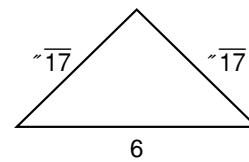**29.** Find the area of this isosceles triangle. Dimensions are in centimeters.
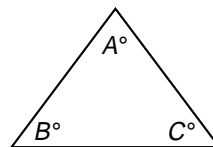

30. Evaluate: $-2^0[-2^0 - 2^2 - (-2)^3 - 2](-2^0 - 2) + x - xy$ if $x = -3$ and $y = -4$

LESSON 35   *Angles in polygons • Inscribed quadrilaterals • Fractional exponents*
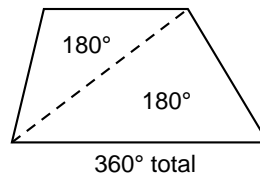
35.A
angles in polygons

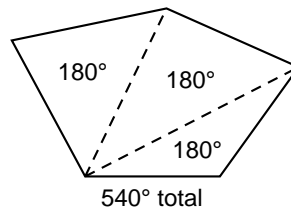We remember that the sum of the interior angles in a triangle is 180°.

$A' + 'B' + 'C' = '180$

A quadrilateral can be triangulated (divided into triangles) from any vertex into two triangles. Thus, the sum of the interior angles of any quadrilateral is 360°.

A pentagon is a 5-gon and can be triangulated from any vertex into three triangles. Thus, the sum of the interior angles of a pentagon is 540°.

    To find the sum of the interior angles of any polygon, we triangulate the polygon from one vertex and multiply the number of triangles by 180°.
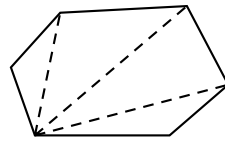
    It is poor practice to memorize a formula when the formula can be developed quickly. If we make a list, we can develop a formula for the sum of the interior angles of any polygon.

$$3 \text{ sides} \longrightarrow 1 \times 180 = 180°$$
$$4 \text{ sides} \longrightarrow 2 \times 180 = 360°$$
$$5 \text{ sides} \longrightarrow 3 \times 180 = 540°$$

Each time we add a side to the polygon, we can draw another triangle.

6 sides → 4 × 180° = 720°

From this we see that the sum of the measures of the interior angles of a polygon of *n* sides is

$$(n - 2) \times 180°$$

We can also find the sum of the measures of the exterior angles. **The sum of the measures of the exterior angles of any polygon is 360°.** We can see this clearly if we let our stick man walk around this polygon.
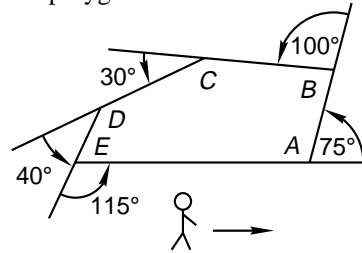
At *A,* he turns through an angle of 75°, and at *B* he turns through an angle of 100°. At *C, D*, and *E*, he turns through angles of 30°, 40°, and 115°. When he gets back to the starting point, he has completed a full circle, and so the sum of these five angles must be 360°. We can see that he will turn through 360° when he walks around any polygon, regardless of the number of sides the polygon has. This shows that the sum of the measures of the exterior angles of any polygon is 360°.

example 35.1   Find *x* and *y*.
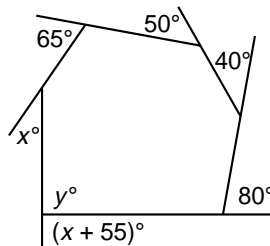
solution   The sum of the measures of the exterior angles of a polygon is 360°. So

$$x + (x + 55) + 80 + 40 + 50 + 65 = 360 \qquad \text{exterior angles} = 360°$$
$$2x + 290 = 360 \qquad \text{added}$$
$$2x = 70 \qquad \text{added } -290 \text{ to both sides}$$
$$\mathbf{x = 35}$$

Since *x* + 55 plus *y* equals a straight angle, or 180°, we can find *y*.

$$x + 55 + y = 180 \qquad \text{equation}$$
$$35 + 55 + y = 180 \qquad \text{substituted}$$
$$\mathbf{y = 90} \qquad \text{solved}$$

35.B

inscribed quadrilaterals

If a quadrilateral is inscribed in a circle, the sum of the measures of any pair of opposite angles is 180°.

To see why this is true, we remember that the measure of an inscribed angle is half the measure of the intercepted arc. In the figure on the left, the measure of angle *A* is half the measure of arc *BCD*.
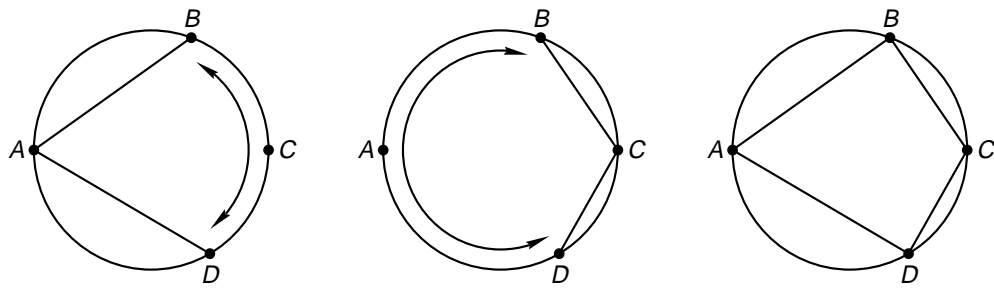
In the center figure, the measure of angle C is half the measure of arc BAD. Since the two arcs go all the way around the circle, the measure of the sum of the two arcs is $360°$. Thus, the sum of the measures of angles A and C is half of $360°$, or $180°$.

example 35.2 Find x, y, and z.
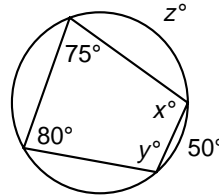
solution This problem gives us practice with the fact that an intercepted arc has twice the measure of the inscribed angle and with the fact that the sum of two opposite angles in an inscribed quadrilateral is $180°$. Also the sum of the measures of the interior angles of any quadrilateral is $360°$. First we note that the arc of the $80°$ angle equals $z° + 50°$. Thus,

$$z + 50 = 2 \times 80 \qquad \text{arc°} = 2 \times \text{(inscribed angle)°}$$

$$z + 50 = 160 \qquad \text{multiplied}$$

$$z = \mathbf{110} \qquad \text{solved}$$

Since opposite angles in an inscribed quadrilateral sum to $180°$, we can solve for x and y.

$$x + 80 = 180 \qquad\qquad\qquad y + 75 = 180$$

$$\text{so} \quad x = \mathbf{100} \qquad\qquad\qquad \text{so} \quad y = \mathbf{105}$$

35.C

fractional exponents

There are two ways to write the square root of 2.

$$\sqrt{2} \qquad \text{and} \qquad 2^{1/2}$$

There is nothing to understand, for this is a definition of what we mean when we use the square root radical sign or the fractional exponent $\frac{1}{2}$. We use the fractional exponent $\frac{1}{3}$ to designate the cube root, the fractional exponent $\frac{1}{4}$ to designate the fourth root, etc.

$$2^{1/2} = \sqrt{2} \qquad 2^{1/3} = \sqrt[3]{2} \qquad 2^{1/4} = \sqrt[4]{2}$$

The rules for exponents are the same for fractional exponents as they are for integral exponents. Thus, both the product theorem and power theorem also apply to fractional exponents.

$$x^{5/3} \cdot x^{8/3} = x^{13/3} \qquad \text{and} \qquad (x^{5/3})^{8/3} = x^{40/9}$$

example 35.3 Simplify: $4^{-1/2}$

solution Negative exponents are not operational indicators, so our first step is to write the expression with a positive exponent. Then we simplify.

$$4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{2}$$

example 35.4   Simplify: $-27^{-1/3}$

solution   If the minus sign is unprotected by a parenthesis, we may cover it up with a finger.

$$\text{👆}\ 27^{-1/3}$$

Now we simplify $27^{-1/3}$.

$$\text{👆}\ 27^{-1/3}\ =\ \text{👆}\ \frac{1}{27^{1/3}}\ =\ \text{👆}\ \frac{1}{3}$$

Now we remove our finger to uncover the minus sign and see that the answer is

$$-\frac{1}{3}$$

example 35.5   Simplify: $16^{3/2}$

solution   This expression can be simplified in two ways.

$$\text{(a)}\quad (16^3)^{1/2}\qquad \text{(b)}\quad (16^{1/2})^3$$

Both (a) and (b) have the same value as the original expression. Next we simplify both expressions within the parentheses and get

$$\text{(a)}\quad (4096)^{1/2}\qquad \text{(b)}\quad (4)^3$$

Both of these have a value of 64, but it is much easier to raise 4 to the third power than it is to take the square root of 4096. For this reason, it is recommended that the fraction always be left inside the parentheses when doing the first step of the simplification. Also we note that it is necessary to learn to recognize that these problems are contrived problems designed to give practice in working with fractional exponents. Not all expressions with fractional exponents can be simplified, for if we have

$$15^{3/2}$$

we can do nothing, for neither

$$(15^{1/2})^3\qquad \text{nor}\qquad (15^3)^{1/2}$$

can be simplified without using logarithms or a calculator.

example 35.6   Simplify: $-8^{-2/3}$

solution   Since the minus sign is not "protected" by a parenthesis, we mentally cover up this sign with a finger and get

$$\text{👆}\ 8^{-2/3}$$

This can be rewritten as

$$\text{👆}\ \frac{1}{8^{2/3}}$$

Next we write the exponent as a product that has the fraction inside the parentheses.

$$\text{👆}\ \frac{1}{(8^{1/3})^2}$$

Now we simplify

$$\text{👆}\ \frac{1}{2^2}$$

and bring back the minus sign to get

$$-\frac{1}{4}$$

practice

a. Find the sum of the interior angles in a 14-gon.

b. Find the sum of the exterior angles in a 14-gon.

Simplify:

c. $64^{-1/2}$ **d.** $64^{2/3}$ **e.** $-125^{-2/3}$

problem set

35

1. Matthew was 1200 yards ahead when Lowe began his pursuit. If Lowe ran 3 times as fast as Matthew and overtook him in 30 minutes, how fast did each boy run?

2. Cheryl and Judy trudged at 4 miles per hour until their packs got too heavy. Then they dropped their packs and continued at the brisk pace of 6 mph. If the total trip of 56 miles took 12 hours, how long did they trudge? How long did they walk briskly?

3. The dhow made the trip in 12 hours while the brigantine made the same trip in 4 hours. If the speed of the brigantine was 6 miles per hour greater than the speed of the dhow, what was the distance traveled by each?

4. Blues were $5 each and yellows cost $8 each. Penelope spent $82, and the number of blues she bought was 2 greater than twice the number of yellows. How many of each kind did she buy?

5. Sixty percent of the aluminum fused as it should have. If 40 tons did not fuse, how much aluminum was there in all?

6. Twenty grams of vanadium was melted with other metals to make 40 grams of the alloy. If 400 grams of the other metals was available, how much vanadium should be used?

7. The figure shown is the base of a right solid that is 10 meters high. Corners that look square are square. Find the lateral surface area of the solid. Di-mensions are in meters.
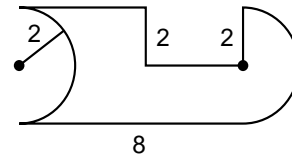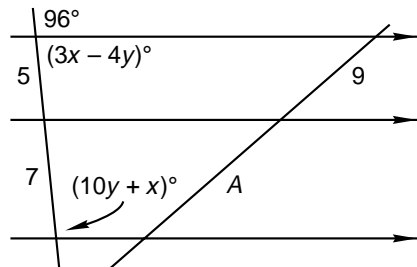
Simplify:

8. $16^{-1/2}$ **9.** $27^{-1/3}$ **10.** $9^{3/2}$ **11.** $-64^{-2/3}$

12. Find x, y, and A.
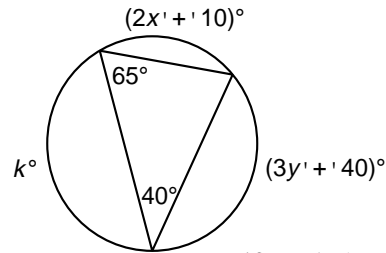
Simplify:

13. $3\sqrt{\dfrac{7}{5}} + 2\sqrt{\dfrac{5}{7}}$ **14.** $2\sqrt{\dfrac{2}{5}} - 9\sqrt{\dfrac{5}{2}}$
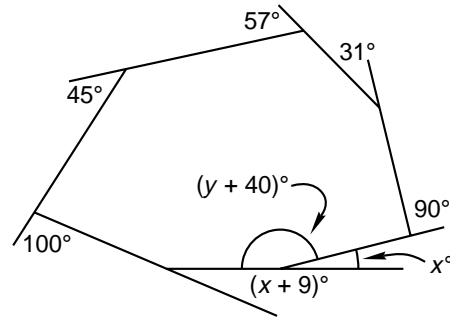
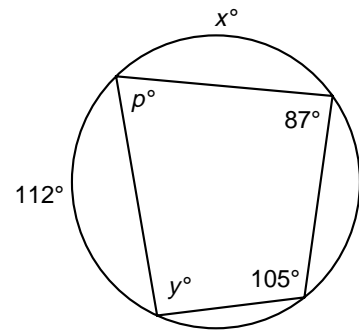**15.** $\dfrac{\dfrac{a}{b} - 4}{\dfrac{xy}{b}}$

16. $\dfrac{\dfrac{x}{x + y} + 6}{\dfrac{4}{x + y}}$

17. Find x, y, and k.

18. Find x. Then find y.

19. Find x, y, and p.

20. Find the equation of the line that passes through the points $(-2, 3)$ and $(4, -5)$.

Add:

21. $\dfrac{4}{x^2(x + y)} + \dfrac{2x - 2}{x(x + y)}$

22. $\dfrac{3x}{x - 2} - \dfrac{2x}{x^2 + x - 6}$

Factor completely:

23. $35a - ax^2 - 2xa$

24. $8x^2 - 2x^3 - x^4$

Simplify:

25. $2\sqrt{3} \cdot \sqrt{12} - 3\sqrt{2} \cdot \sqrt{6} + 4\sqrt{2}(3\sqrt{2} - \sqrt{6})$

26. $\dfrac{(7000 \times 10^{14})(0.0002 \times 10^{-11})}{1400 \times 10^{-10}}$

Solve:

27. $\dfrac{x - 3}{7} - \dfrac{2x}{4} = 5$

28. $0.002x = 0.02 + 0.04$

29. $2\dfrac{1}{3}x - 2x^0 = 3\dfrac{1}{4}$

30. Find the distance between $(-4, 2)$ and $(7, 3)$.